Compression in Self-Organizing Particle Systems

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BARRETT UNDERGRADUATE HONORS THESIS

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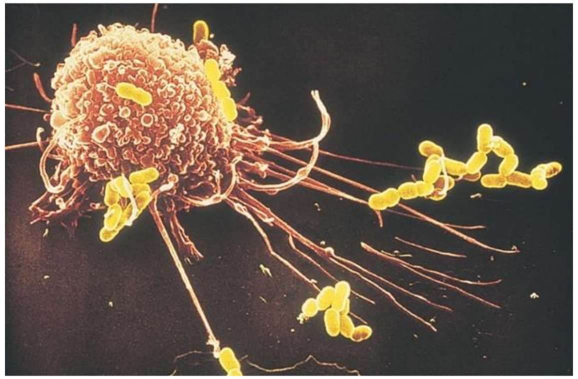
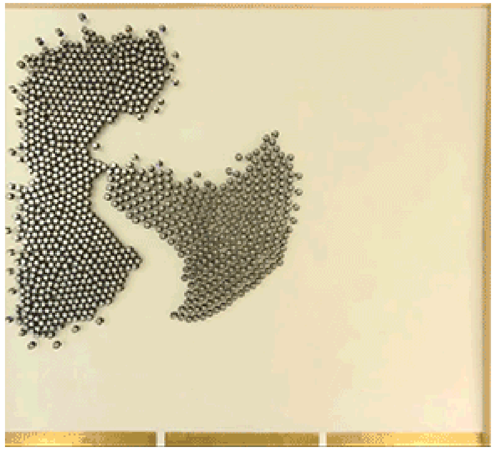
Motivation

Inspirations & Applications



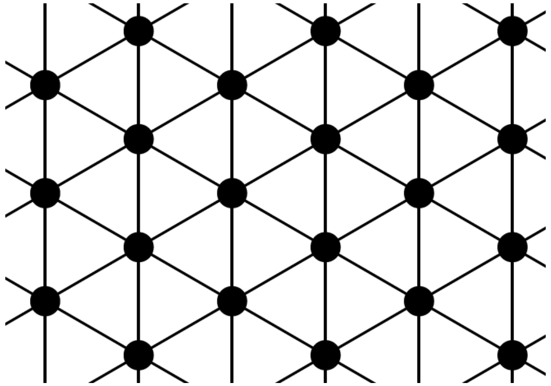
Current Programmable Matter

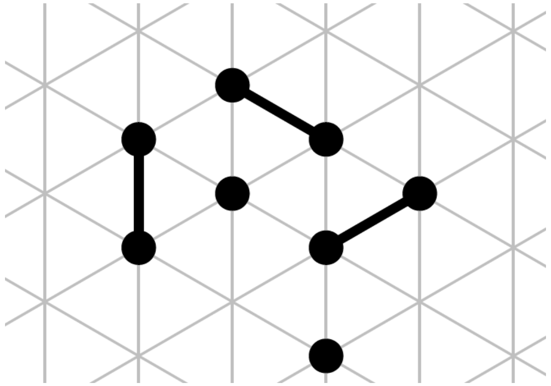


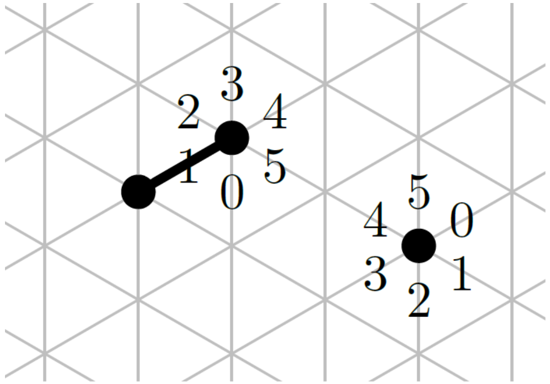
The Amoebot Model

Particles are:

- Anonymous (no unique identifiers)
- Without global orientation or compass (no shared sense of "north")
- Limited in memory (constant size)
- Activated asynchronously





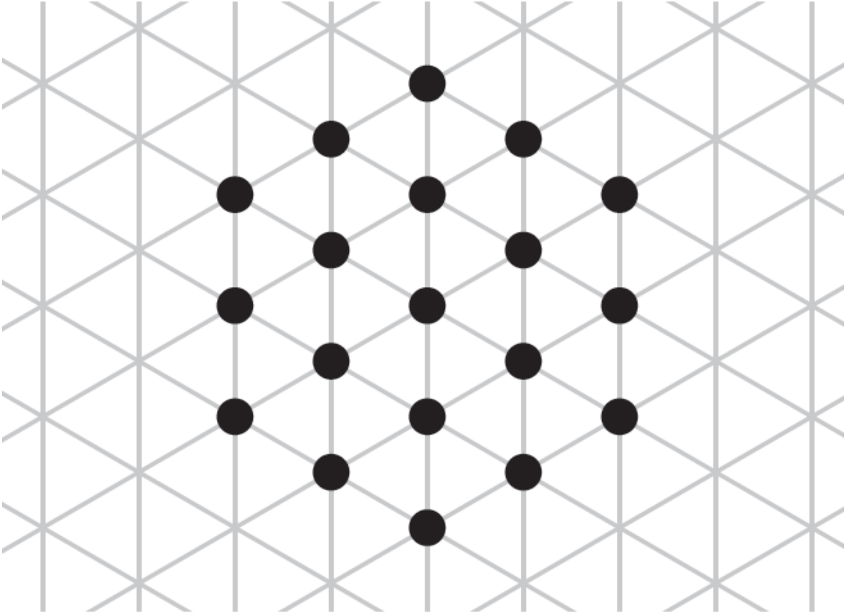


The Compression Problem

Problem: Given a particle system that is initially connected, "gather" the particles as tightly as possible.

Many possible formal interpretations of this:

- Minimize the diameter?
- Minimize the perimeter?
- Maximize the total number of edges formed?
- Maximize the number of induced triangles?
- Eliminate all holes?



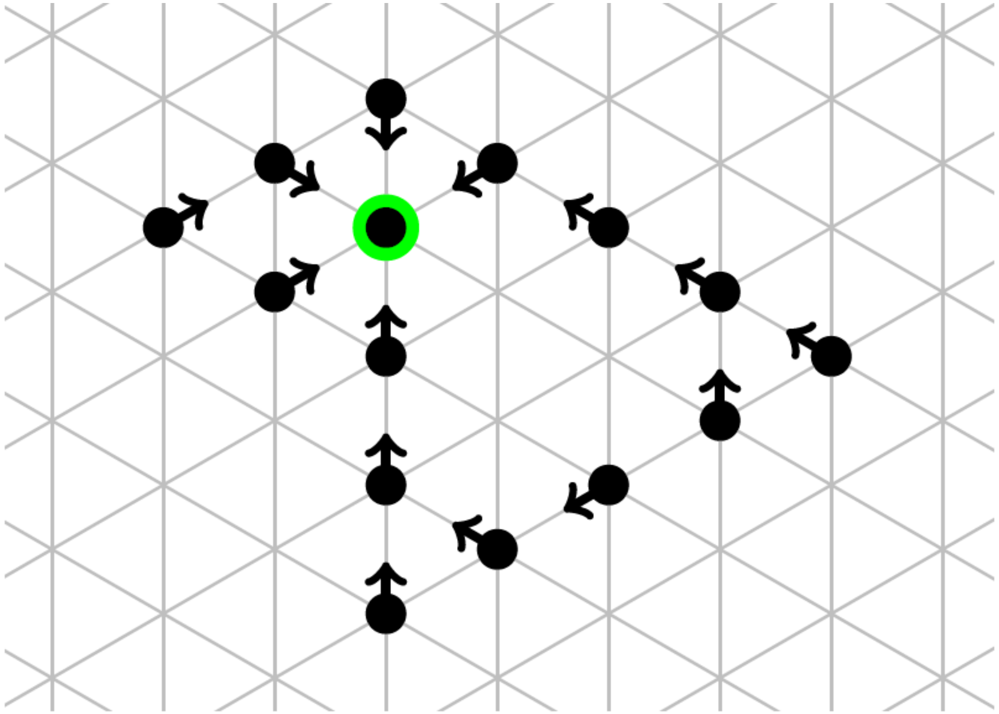
Overview

Two early approaches:

1. *Local Compression*: particles satisfy local rules to achieve global structure
2. *Hole Elimination*: particles detect and eliminate holes contained in their structure

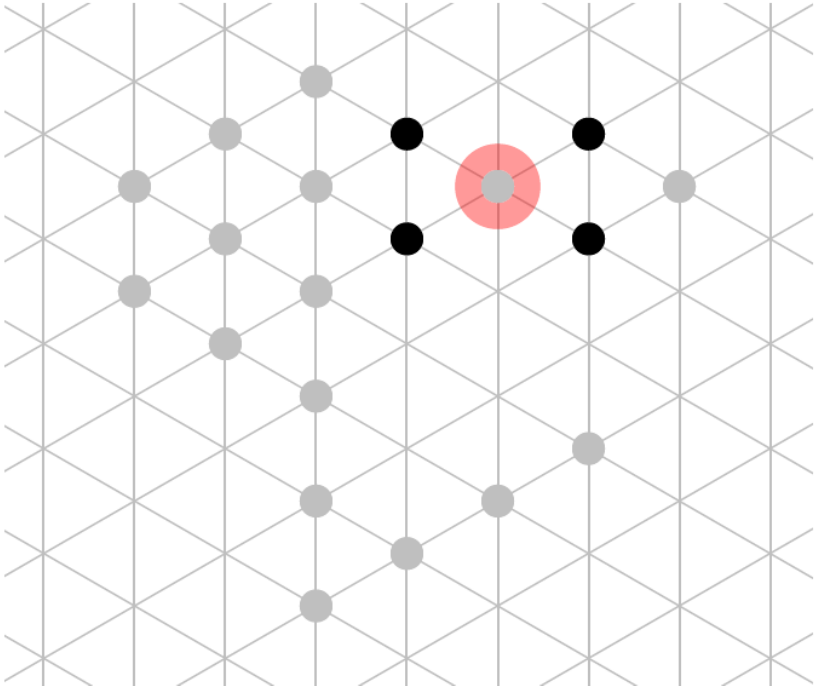
Spanning Forest Primitive

Gives particles a sense of orientation that otherwise does not exist.



Goal

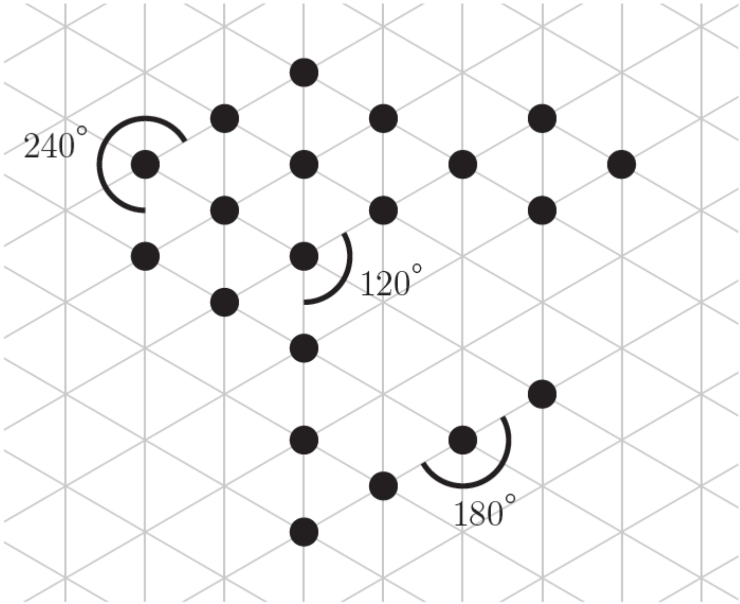
Definition: A particle system is said to be *locally compressed* if every particle *p* is *particle compressed*; that is, (1) *p* does not have exactly five neighbors, and (2) the graph induced by *N(p)* is connected.



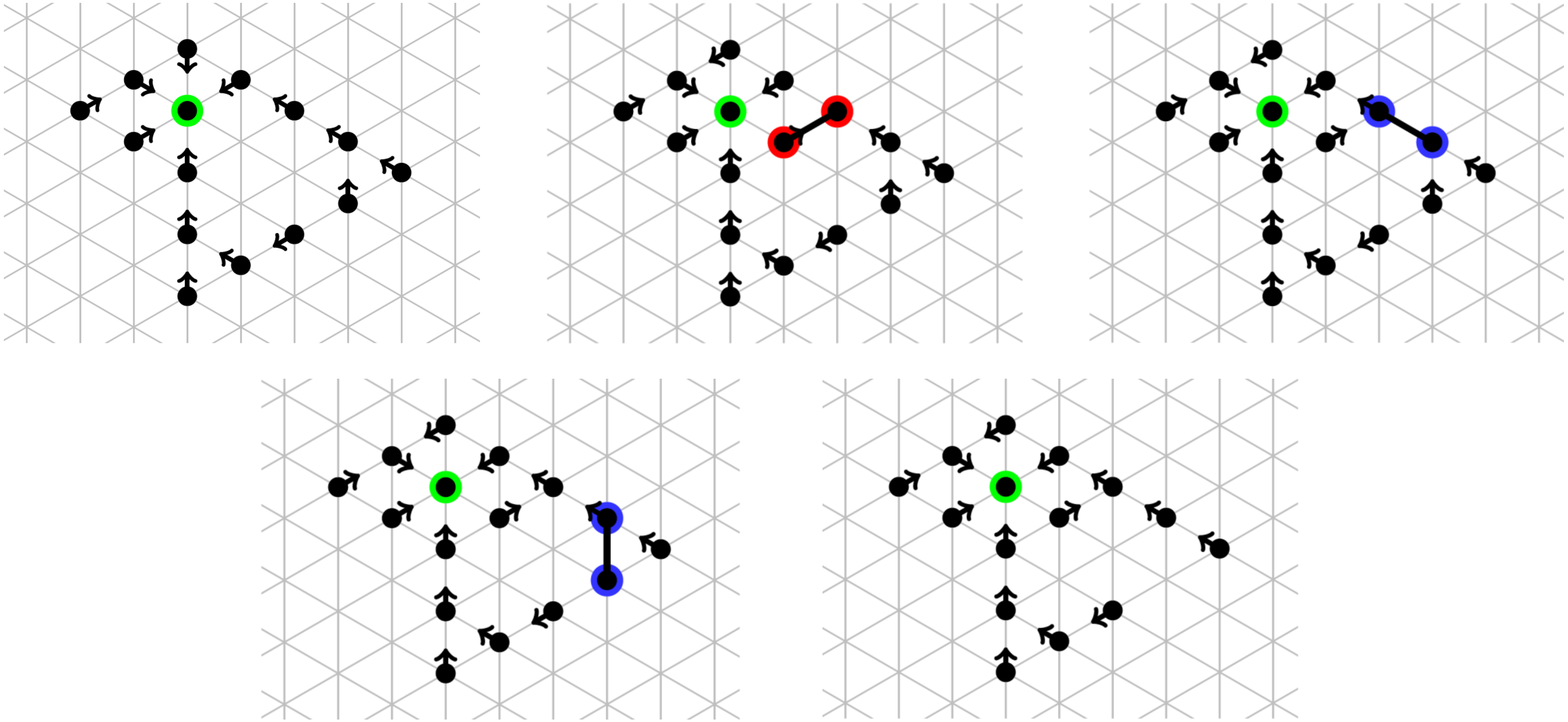
Goal (cont.)

Definition: A particle system is said to be *convex* if every external angle α on the outer border of the system has α ≥ 180°.

Lemma: If a particle system is locally compressed, then it forms a convex configuration containing no holes.

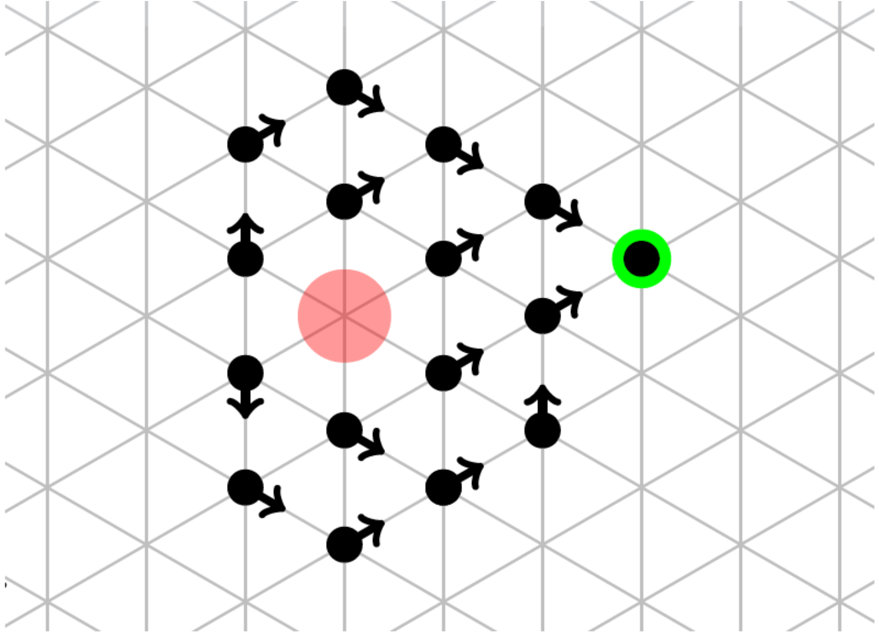


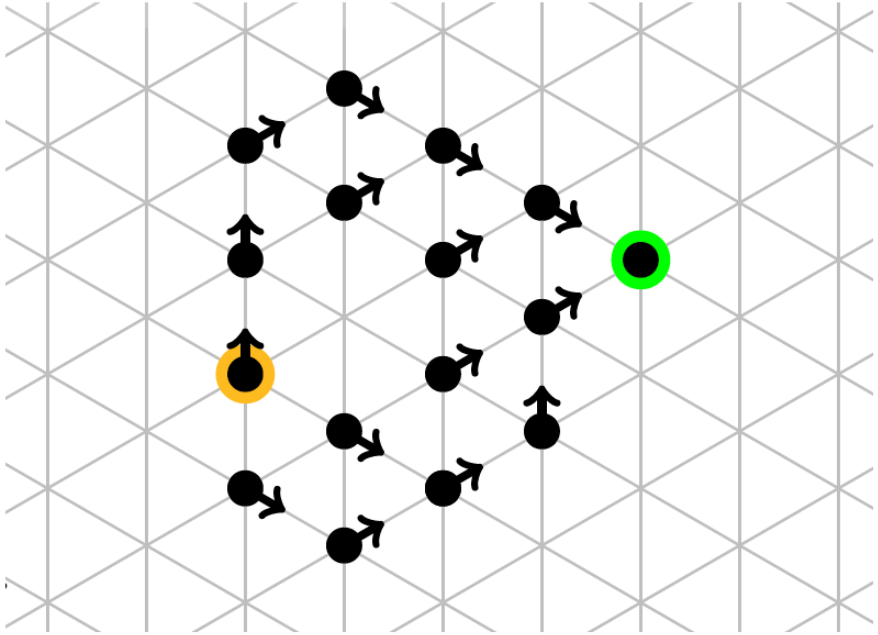
Algorithm Description



Algorithm Description (cont.)

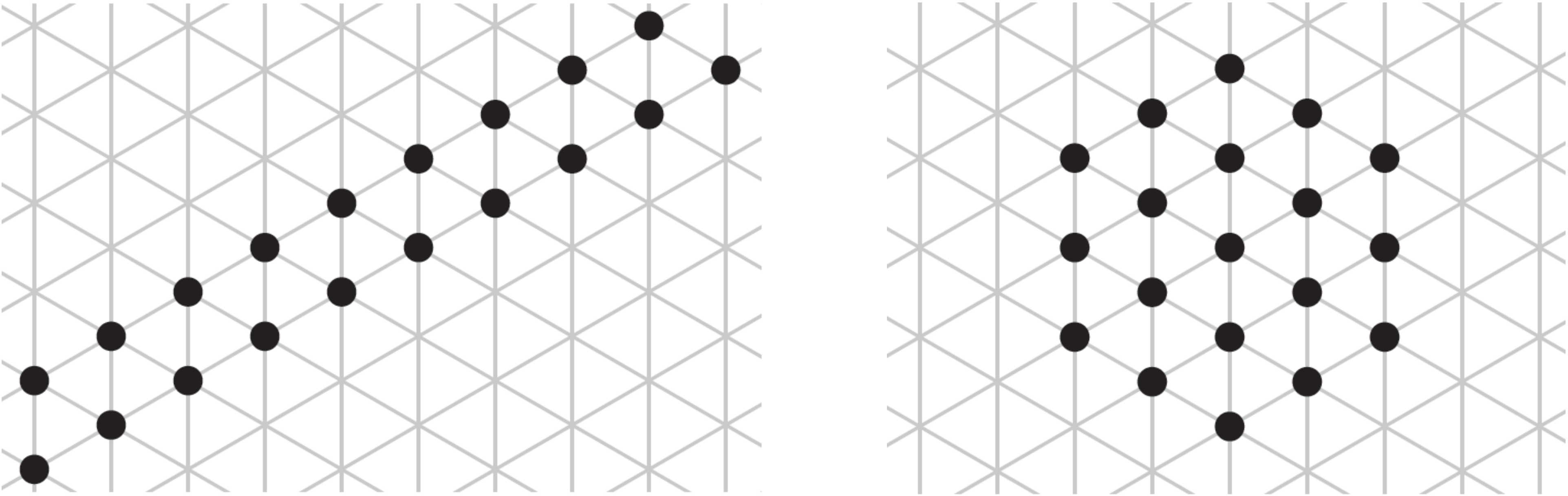
Leaf switching is required when a hole is bounded entirely by leaves.





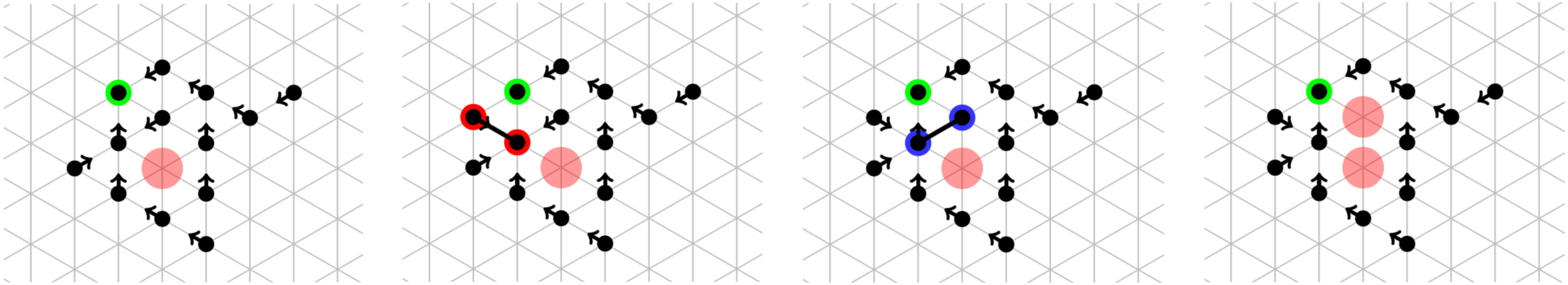
Results

When compression is interpreted as convex and containing no holes, it is too permissive.



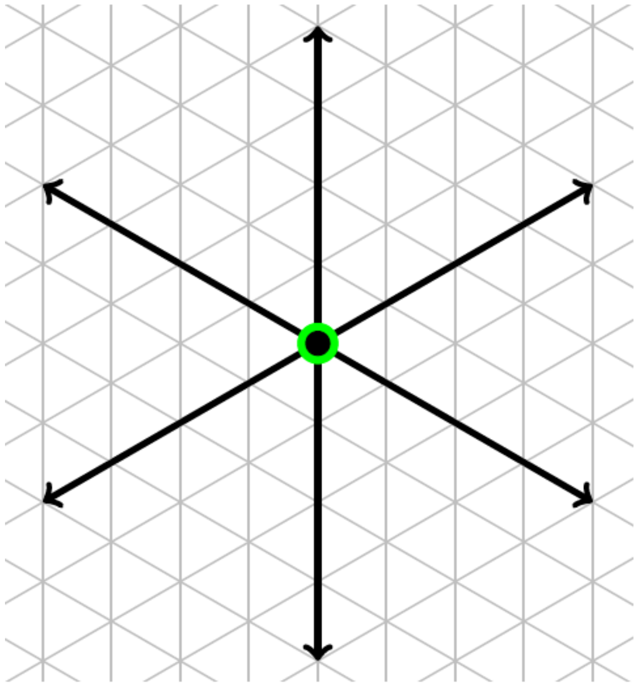
Results (cont.)

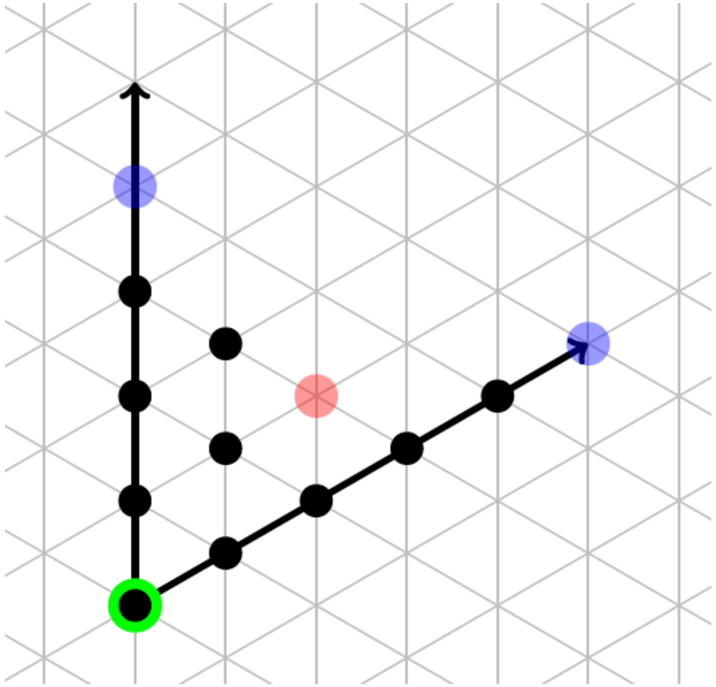
The Local Compression algorithm has the tendency to oscillate ad infinitum.



Goal

Definition: A particle system is said to have achieved *hole elimination* if it contains no holes.





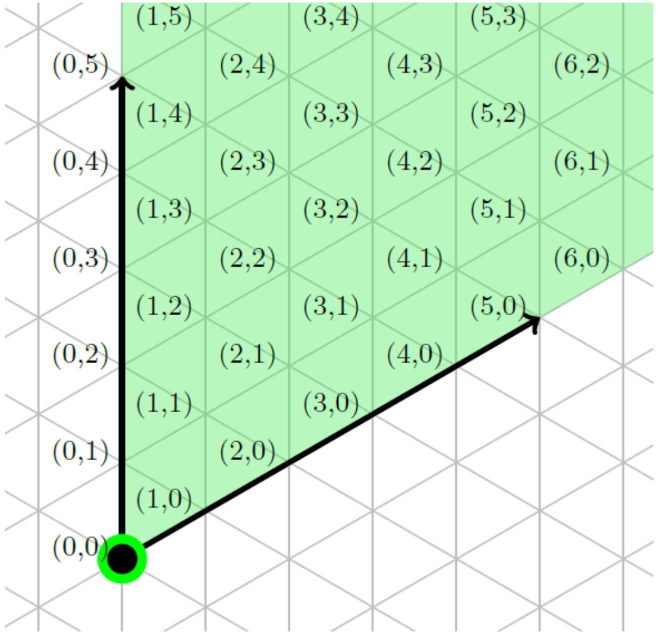
Algorithm Description

[Refer to SOPS Simulator for live demo.]

Correctness Results

Definition: For any wedge *W*, the coordinate system *C* : { (x,y) : x,y ∈ ℕ} → *W* is defined as in the figure below. We define a relation ≤ on the locations (x,y), (x',y') ∈ *C* as follows:

$$(x,y) \leq (x',y') \leftrightarrow x \leq x' \wedge y \leq y'.$$

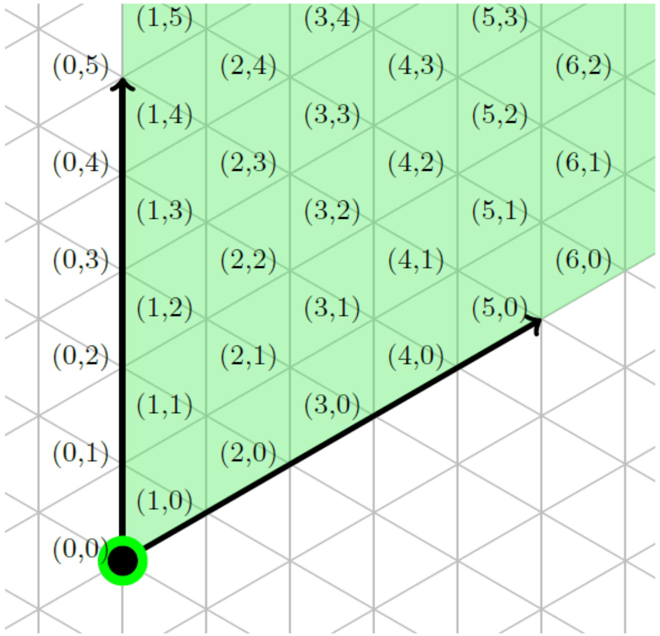


Correctness Results

Lemma: If a location (x,y) in a wedge *W* is docking, then every location (x',y') such that (x',y') < (x,y) is occupied by a finished particle.

Theorem: A particle system entirely composed of finished particles contains no holes.

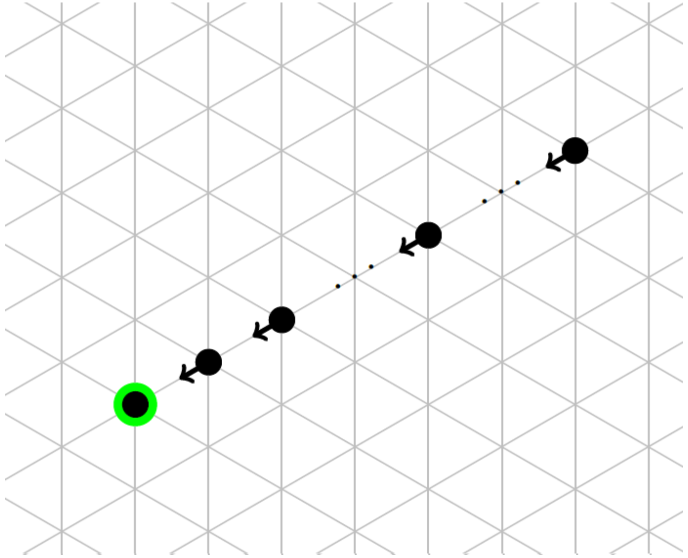
Proof sketch. If a hole did exist, then there exists a location (x,y) in the hole such that (x+1,y) is occupied by a finished particle, contradicting Lemma 2. Therefore, the hole could not have existed in the first place.

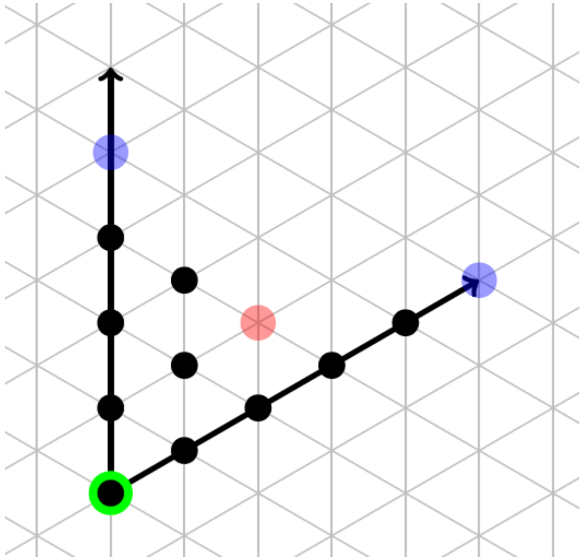


Convergence Results

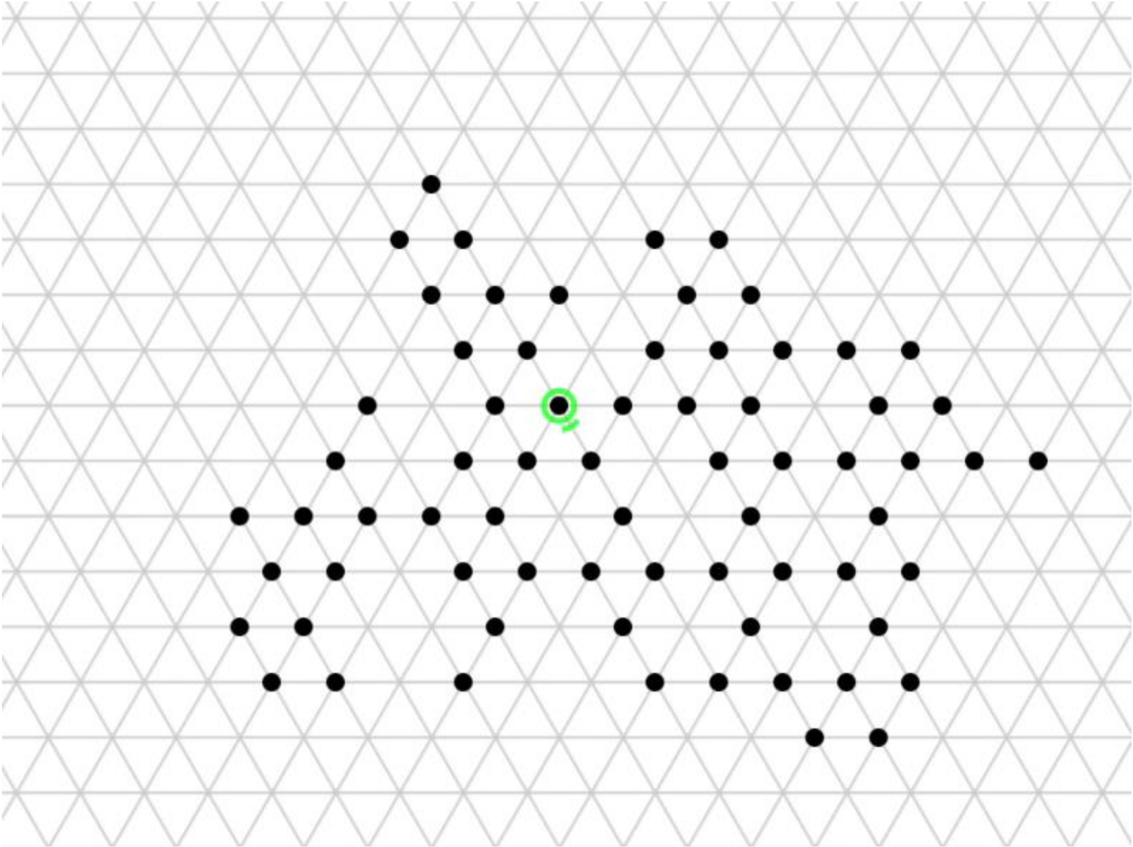
Lemma: Every particle in a spanning tree of size k will become either finished or walking after O(k) rounds.

Theorem: Hole Elimination terminates in the worst case of Θ(n) rounds, where n is the number of particles in the system.

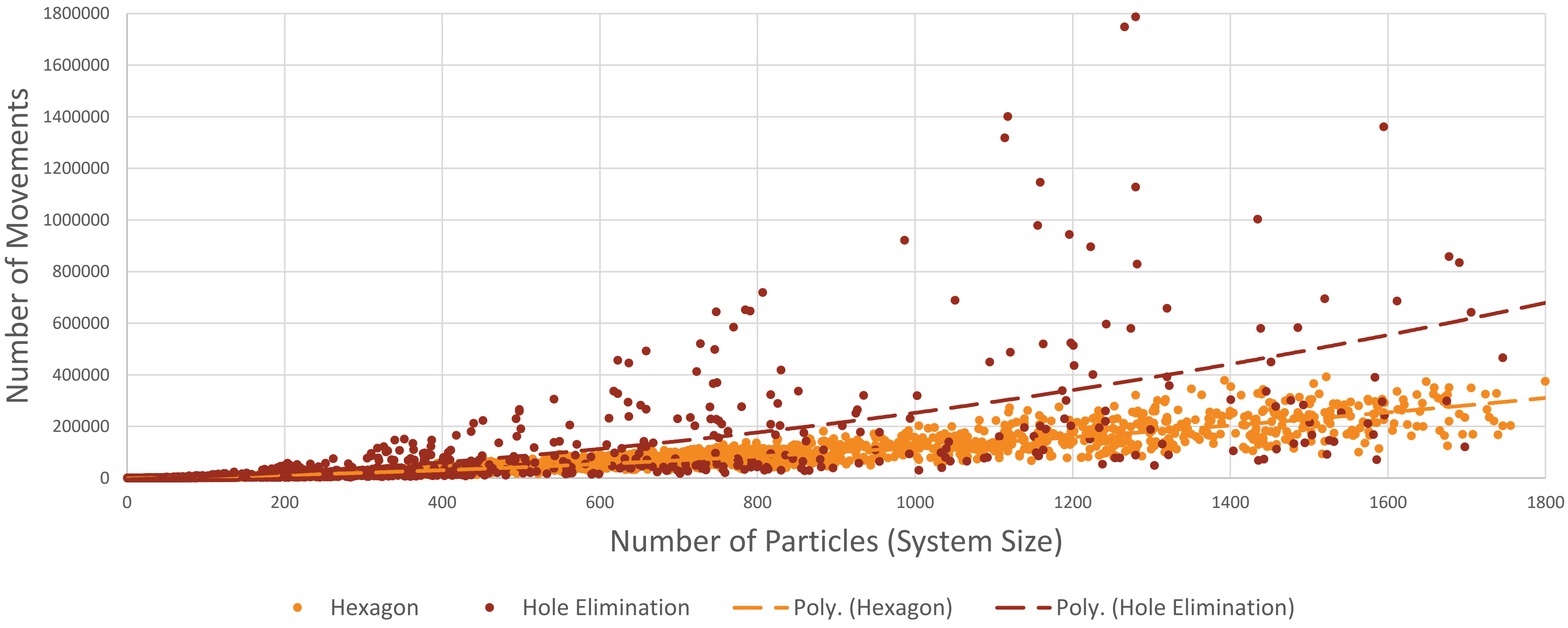




Competitive Analysis



Competitive Analysis (cont.)



Goal

Definition: Given an $\alpha > 1$, a connected configuration σ on n particles is said to be *α-compressed* if $p(\sigma) \leq \alpha \bullet p_{min}(n)$.

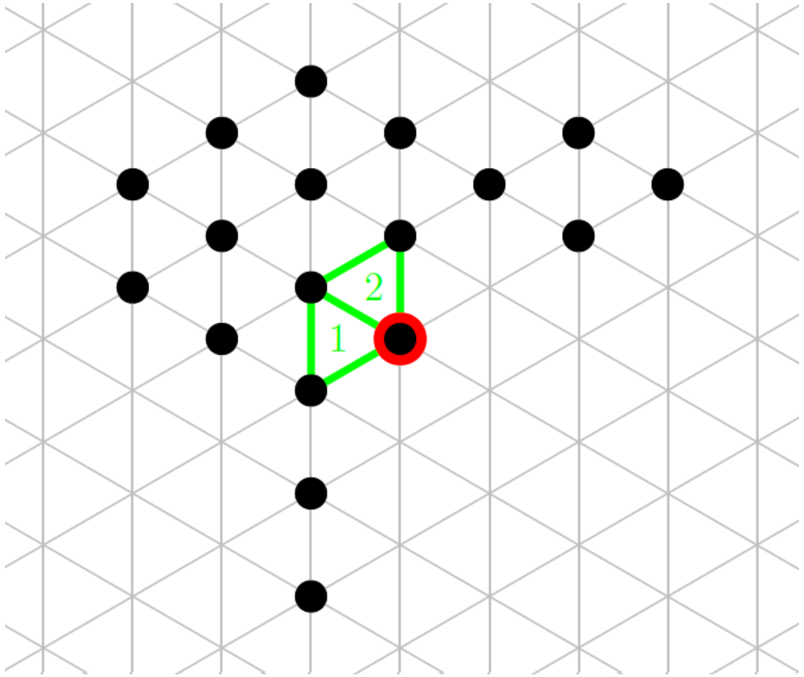
Definition: Given an $0 < \alpha < 1$, a connected configuration σ on n particles is said to be *α-expanded* if $p(\sigma) \geq \alpha \bullet p_{max}(n)$.

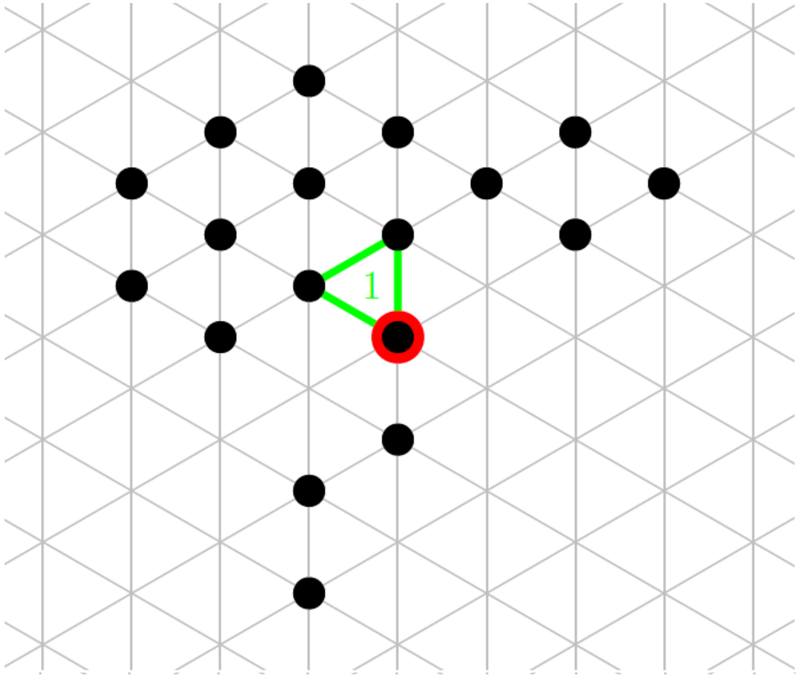
Lemma: For a connected configuration σ on n particles which contains no holes, the number of triangles $t(\sigma) = 2n - p(\sigma) - 2$.

Corollary: $t(\sigma)$ is maximized when $p(\sigma) = p_{min}(n)$.

Markov Chain *M*

Input is a starting configuration σ_0 which is connected and contains no holes, and a bias parameter $\lambda > 1$. Choices are made with probability $\lambda^{t'-t}$.





Results

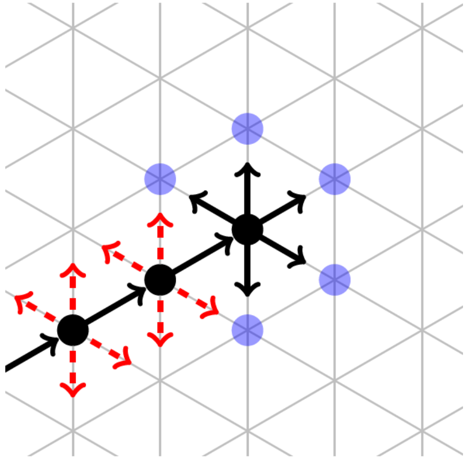
Theorem: Markov chain *M* is *ergodic*, meaning it is *irreducible*—that is, for any configurations x,y there exists a t such that $P^t(x,y) > 0$—and *aperiodic*, that is, for any configurations x,y the g.c.d. $\{ t : P^t(x,y) > 0 \} = 1$.

Theorem: The stationary distribution π of *M* is given by

$$\pi(\sigma) = \frac{\lambda^{t(\sigma)}}{Z} = \frac{\lambda^{-p(\sigma)}}{Z'}, \text{ where } Z = \sum_\sigma \lambda^{t(\sigma)} \text{ and } Z' = \sum_\sigma \lambda^{-p(\sigma)}.$$

Results (cont.)

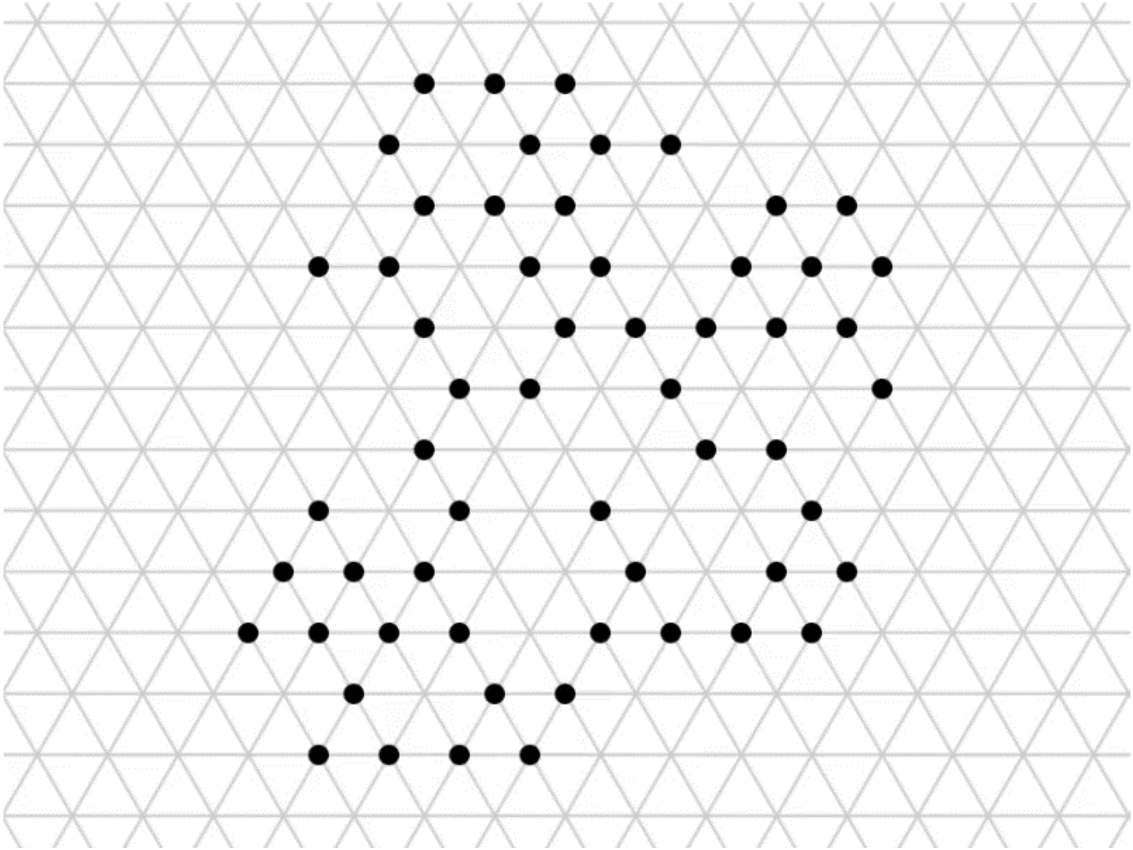
Lemma: The number of connected configurations with no holes and perimeter k is at most 5^k.



Theorem: For any α > 1, there exists a $\lambda^* = 5^{a/(a-1)} > 5$, $n^* \geq 0$, and γ < 1 such that for all $\lambda > \lambda^*$ and $n > n^*$, the probability that a random sample σ drawn according to the stationary distribution π of *M* is not α-compressed is exponentially small:

$$\mathbb{P}(p(\sigma) \geq \alpha \bullet p_{min}(n)) < \gamma^{\sqrt{n}}.$$

Obtaining a Seed



Future Work

For the Markov chain algorithm for compression:

- Further improve the bounds for λ in search of a critical value λ_c.
- Proofs of time complexity using distributed computing techniques.

For the problem of compression in general:

- Generalize to higher dimensions (3D is practical)

References

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3. Sarah Cannon, Joshua J. Daymude, Dana Randall, and Andrea W. Richa. A markov chain algorithm for compression in self-organizing particle systems. CoRR, abs/1603.07991, 2016.

4. Zahra Derakhshandeh, Robert Gmyr, Thim Strothmann, Rida A. Bazzi, Andrea W. Richa, and Christian Scheideler. Leader election and shape formation with self-organizing programmable matter. In *DNA Computing and Molecular Programming – 21st International Conference, DNA 21, Boston and Cambridge, MA, USA, August 17-21, 2015. Proceedings,* pages 117-132, 2015.

5. Zahra Derakhshandeh, Robert Gmyr, Andrea W. Richa, Christian Scheideler, and Thim Strothmann. An algorithmic framework for shape formation problems in self-organizing particle systems. In *Proceedings of the Second Annual International Conference on Nanoscale Computing and Communication, NANOCOM'15, Boston, MA, USA, September 21-22, 2015,* pages 21:1-21:2, 2015.

Thank you!