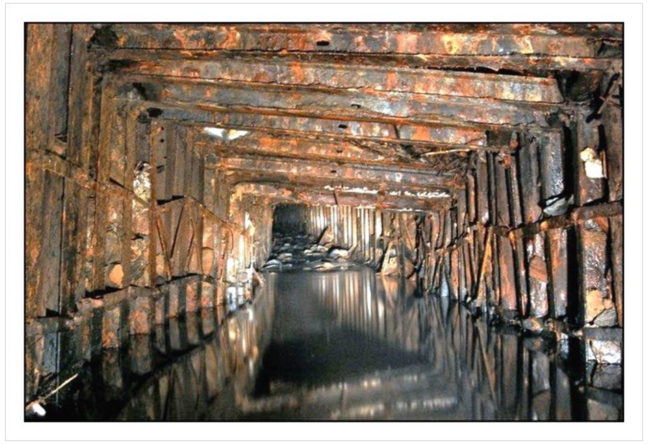
Local Stochastic Algorithms for Compression and Shortcut Bridging in Programmable Matter

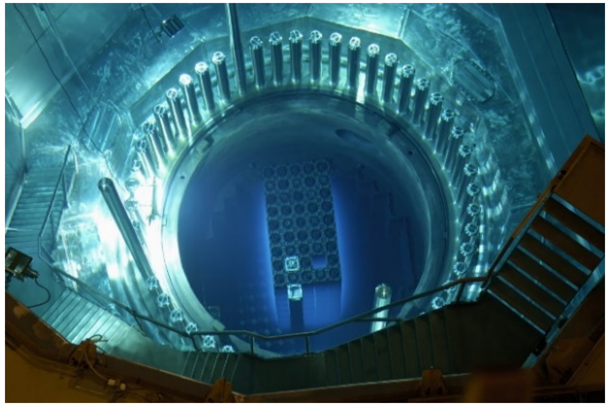
JOSHUA J. DAYMUDE AND ANDRÉA W. RICHA – ARIZONA STATE UNIVERSITY

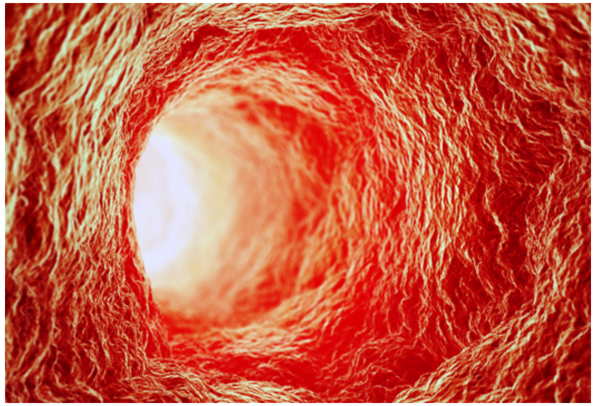
SARAH CANNON AND DANA RANDALL – GEORGIA INSTITUTE OF TECHNOLOGY

MARTA ANDRÉS ARROYO – UNIVERSITY OF GRANADA

Inspirations & Applications







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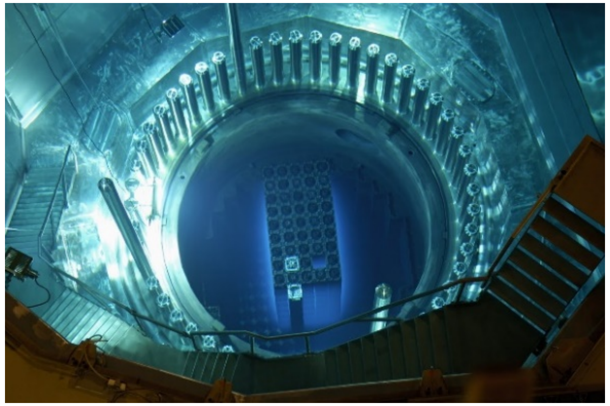
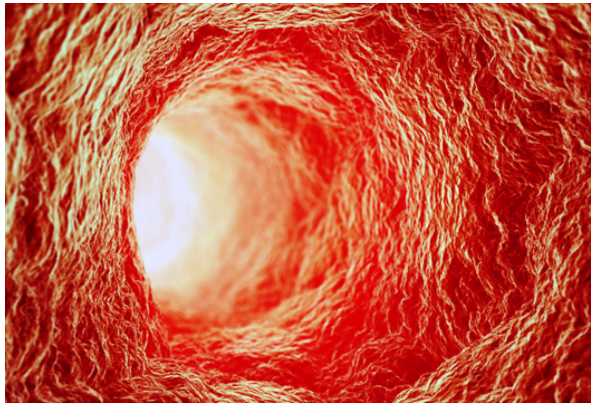
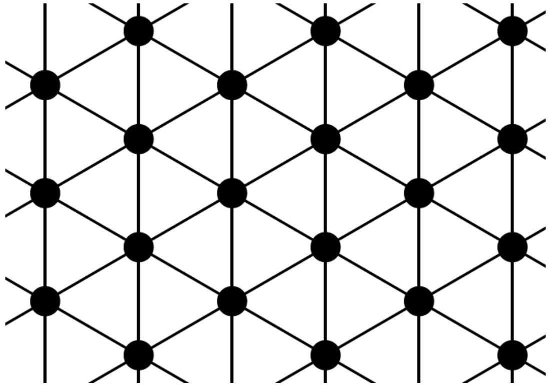


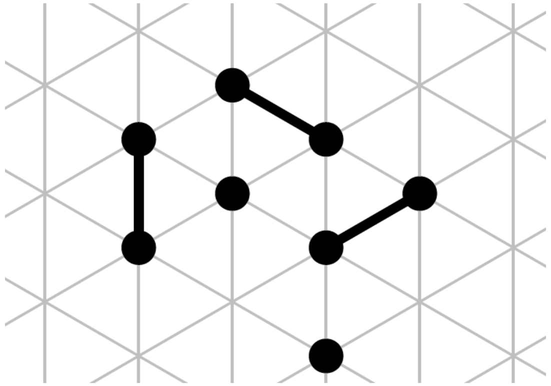


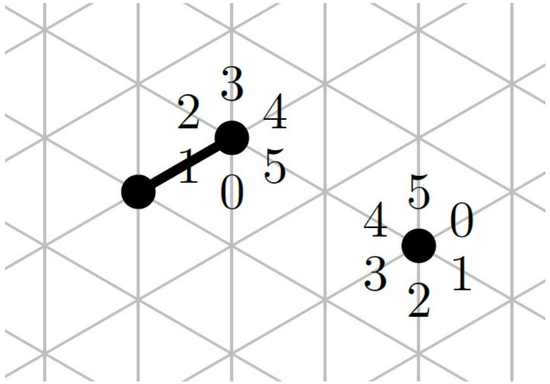
The Amoebot Model

Particles move by *expanding* and *contracting,* and are:

- Anonymous (no unique identifiers)
- Without global orientation or compass (no shared sense of “north”)
- Limited in memory (constant size)
- Activated asynchronously

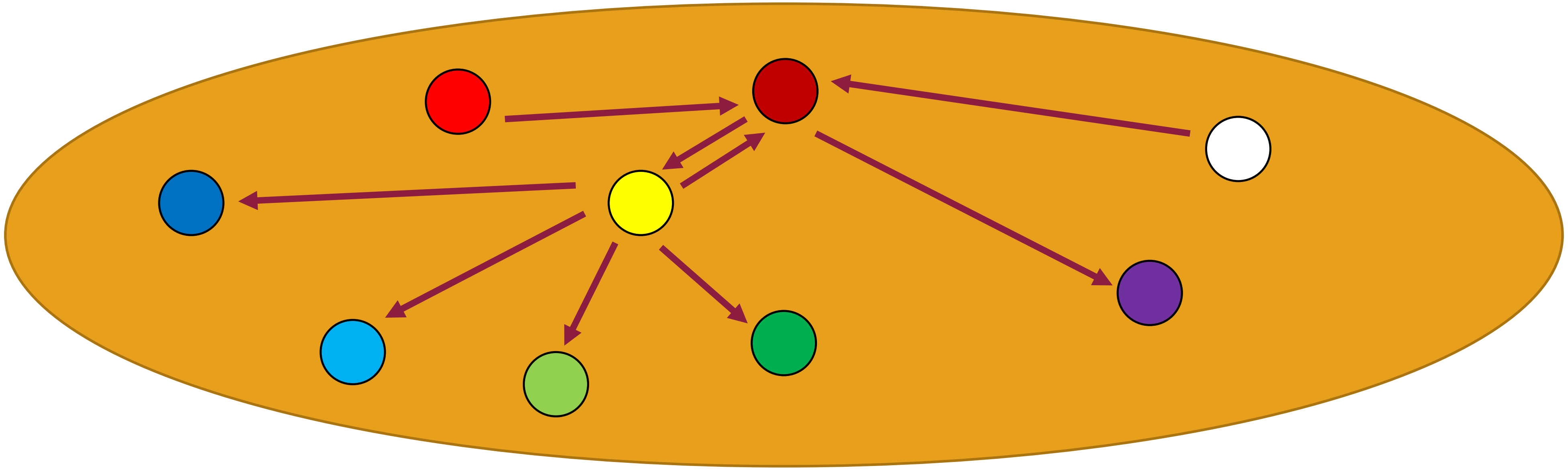






Markov Chains

- A *Markov chain* is a memoryless random process that undergoes transitions between states in some state space.



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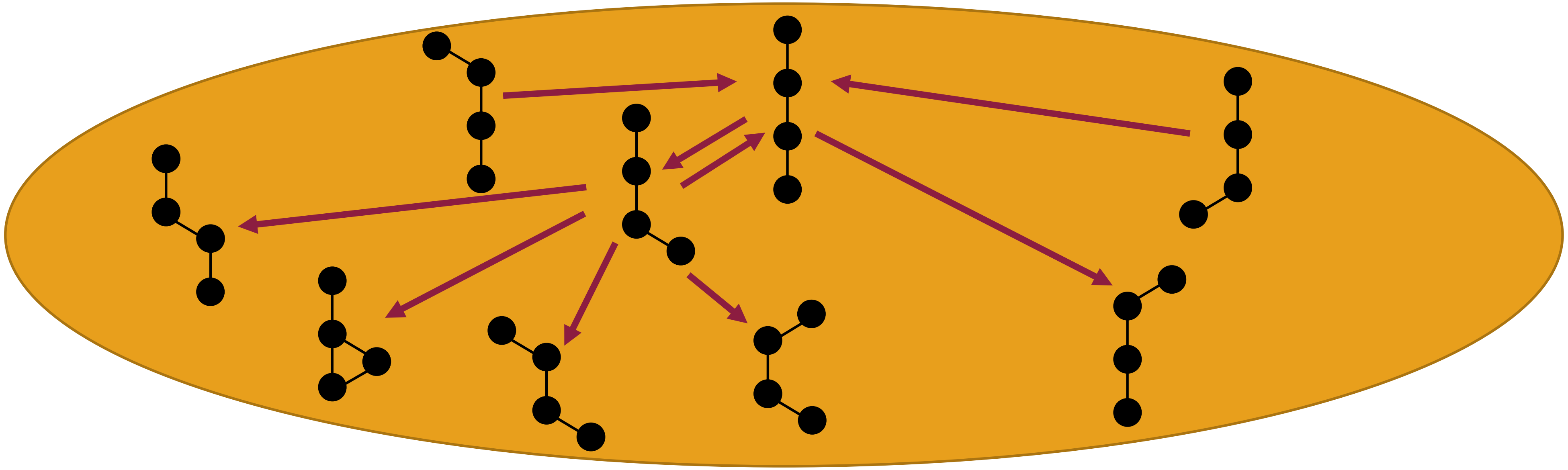
- In our context, states are *particle system configurations*, and transitions between them are individual particle movements.



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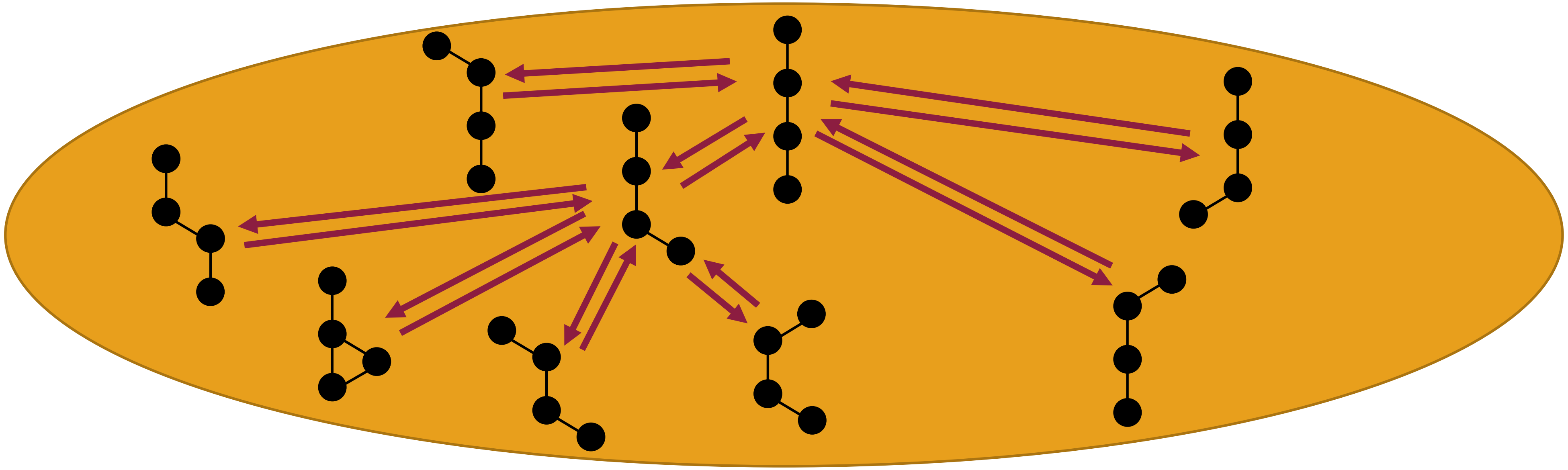


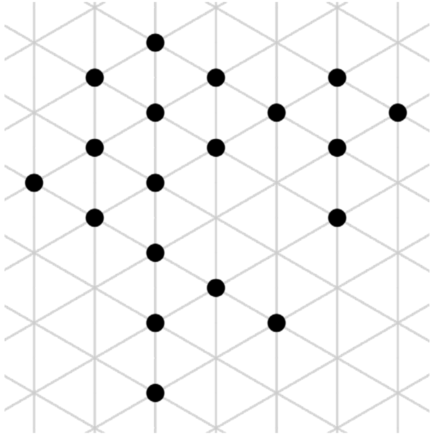
The Compression Problem

Informally: Gather a particle system *P* as tightly together as possible.

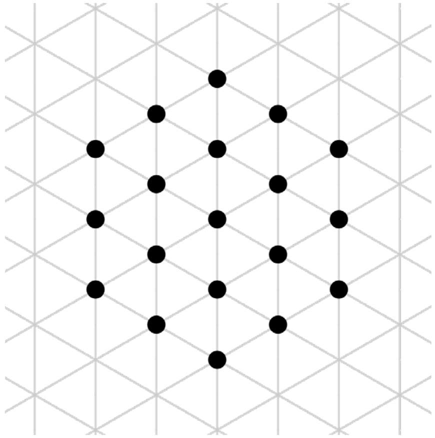
Formally:

- The ***perimeter*** of a connected, hole-free configuration σ, denoted p(σ), is the length of σ's outer boundary. Let p_{min} denote the minimum possible perimeter.
- Given a constant α > 1, σ is said to be ***α-compressed*** if p(σ) ≤ α · p_{min}.

"2-compressed"
p(σ) = 23
p_{min} = 12

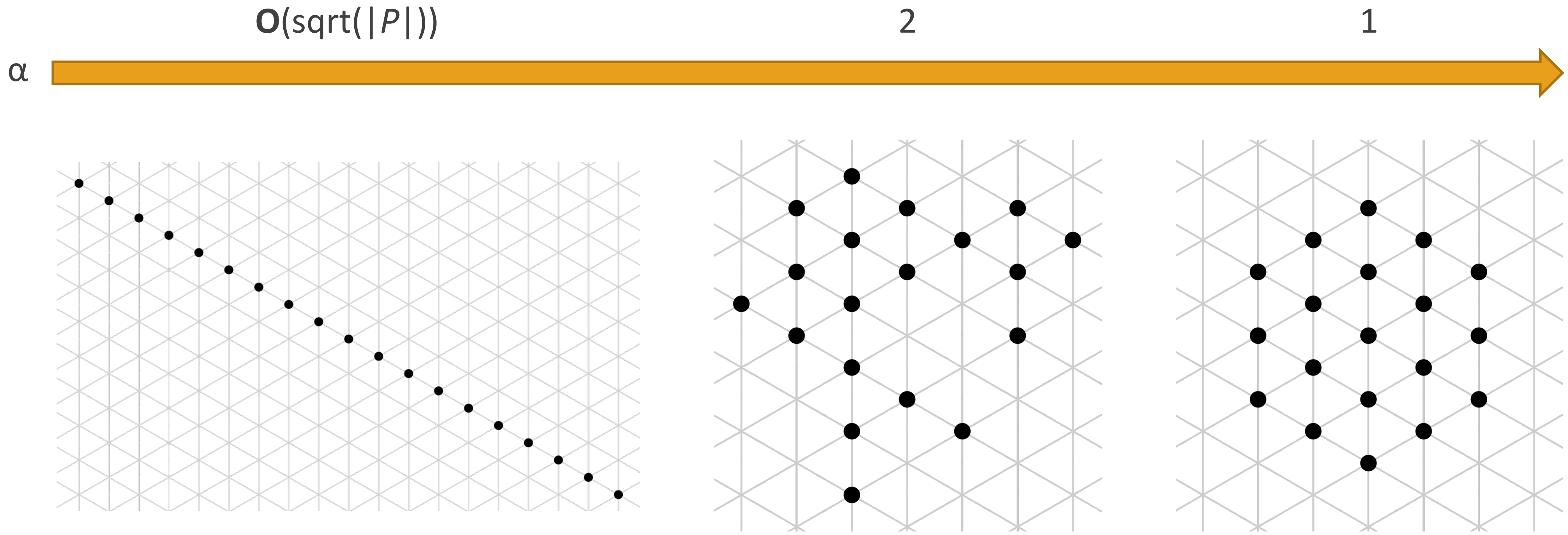


"1-compressed"
p(σ) = 12
p_{min} = 12



Our Goal

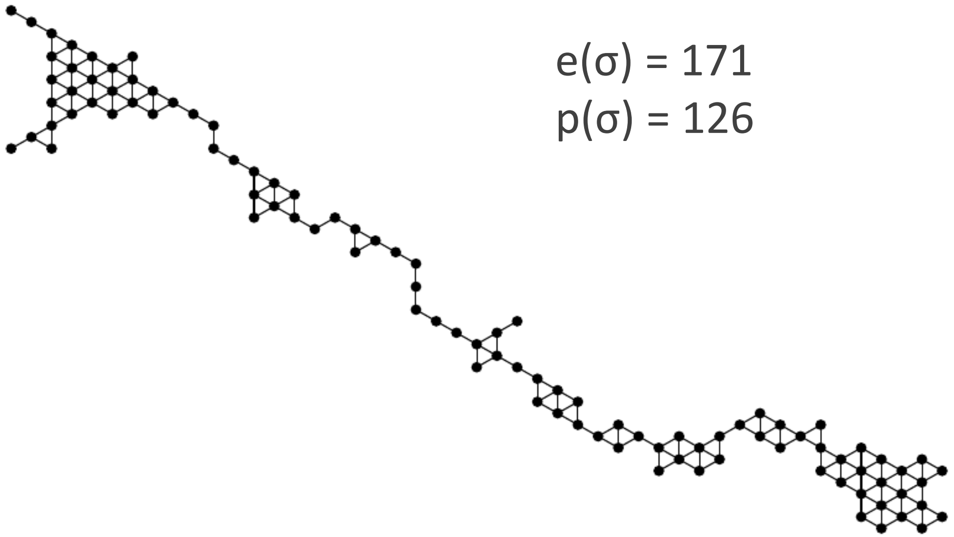
- **Goal 1:** Given a particle system *P* and a constant α > 1, reach and remain in a set of configurations which are α-compressed.



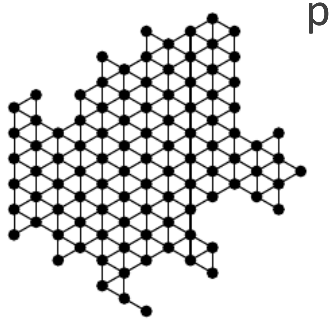
Translating Global To Local

Perimeter is a *global* property, but our particles are limited to *local* communication.

- **Lemma:** Maximizing the number of internal edges is equivalent to minimizing perimeter.
- First attempt: particles move to positions where they have more neighbors.



$e(\sigma) = 171$
$p(\sigma) = 126$

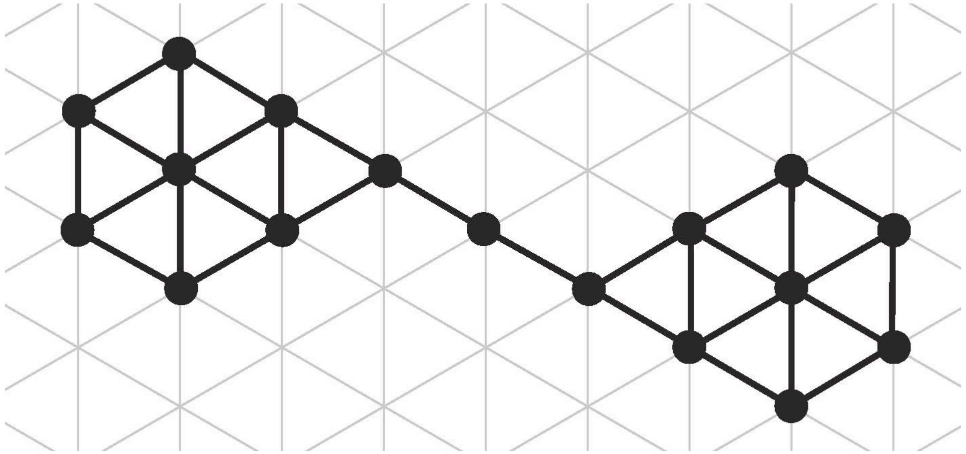


$e(\sigma) = 246$
$p(\sigma) = 51$

Translating Global To Local

Perimeter is a *global* property, but our particles are limited to *local* communication.

- **Lemma:** Maximizing the number of internal edges is equivalent to minimizing perimeter.
- First attempt: particles move to positions where they have more neighbors.
- …However, need something more robust to local minima.



Markov Chains for Particle Systems

The general framework:

1. Choose a particle from the system uniformly at random.
2. Choose a direction from {0, … , 5} and a number p from (0,1) uniformly at random.
3. If certain properties hold and p < [probability function], then move in that direction.
4. Otherwise, do nothing.

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4. Otherwise, do nothing.

These are customizable for different applications!

A Markov Chain for Compression

Input: an initial configuration σ_0 (connected, hole-free), and a bias parameter $\lambda > 1$.

1. Choose a particle from the system uniformly at random.
2. Choose a direction from {0, … , 5} and a number p from (0,1) uniformly at random.
3. If properties for maintaining connectivity and avoiding holes hold and $p < \lambda^{\Delta e}$, then move in that direction.
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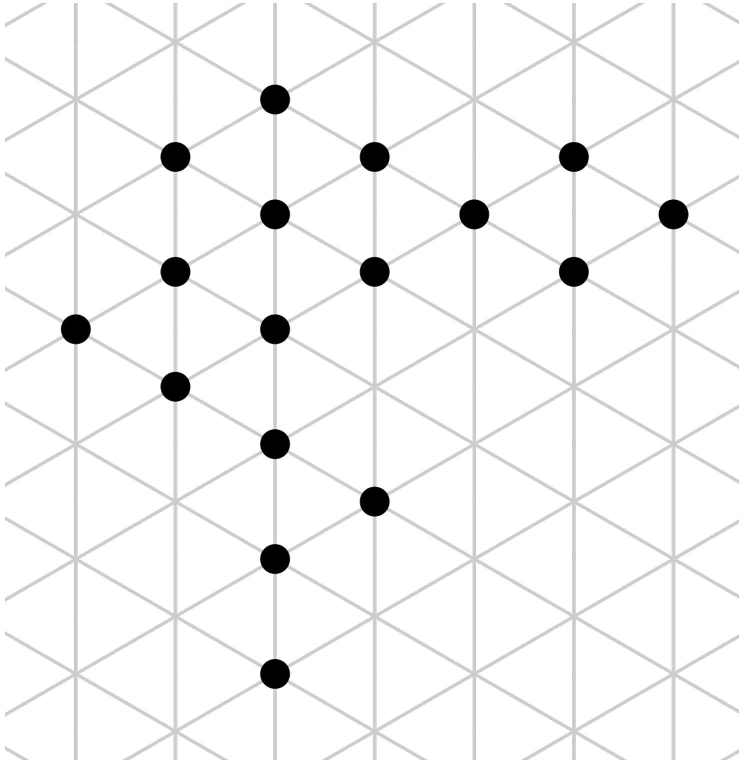
A Markov Chain for Compression

Recall: movement decisions are made with probability $\lambda^{\Delta e}$, where $\lambda > 1$.



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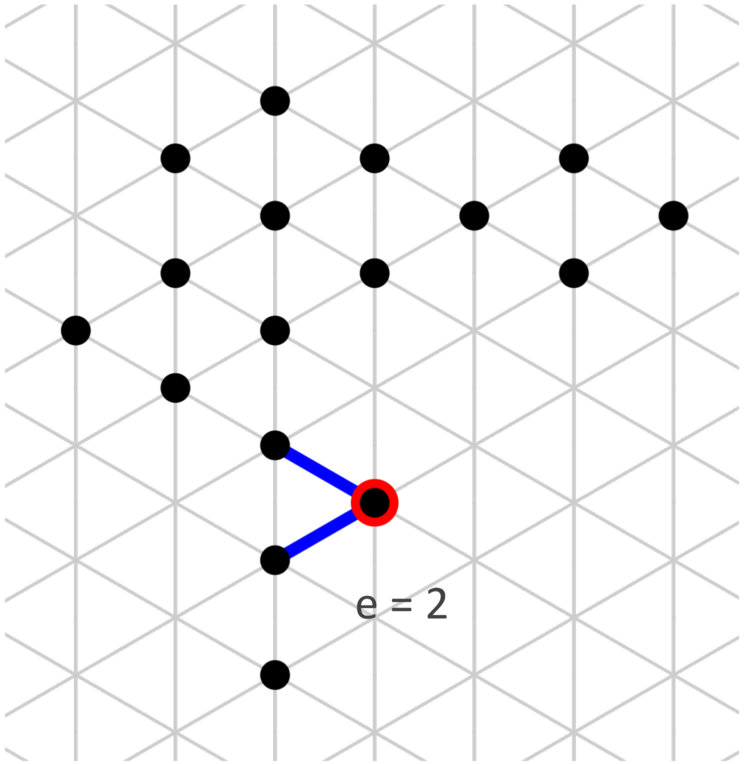

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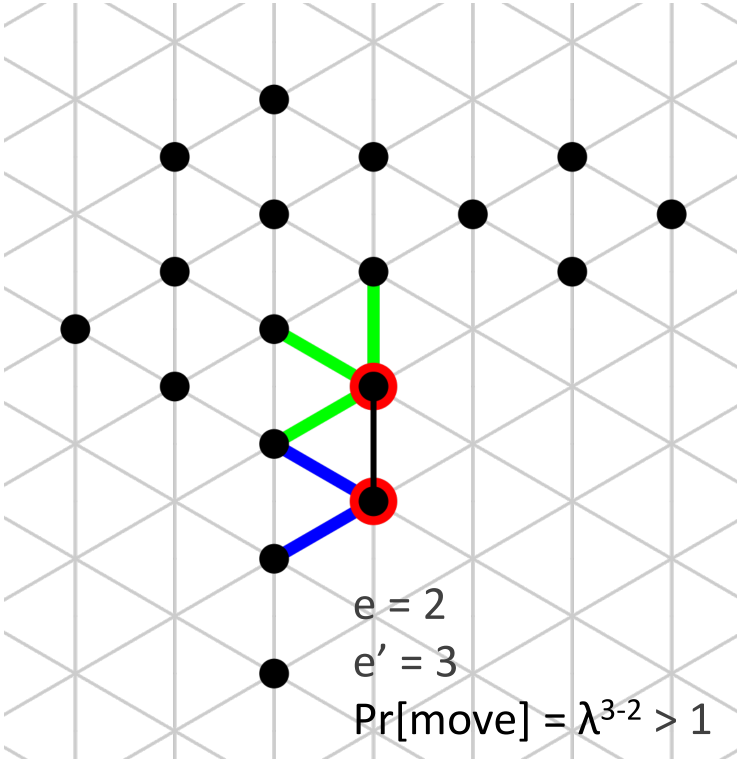

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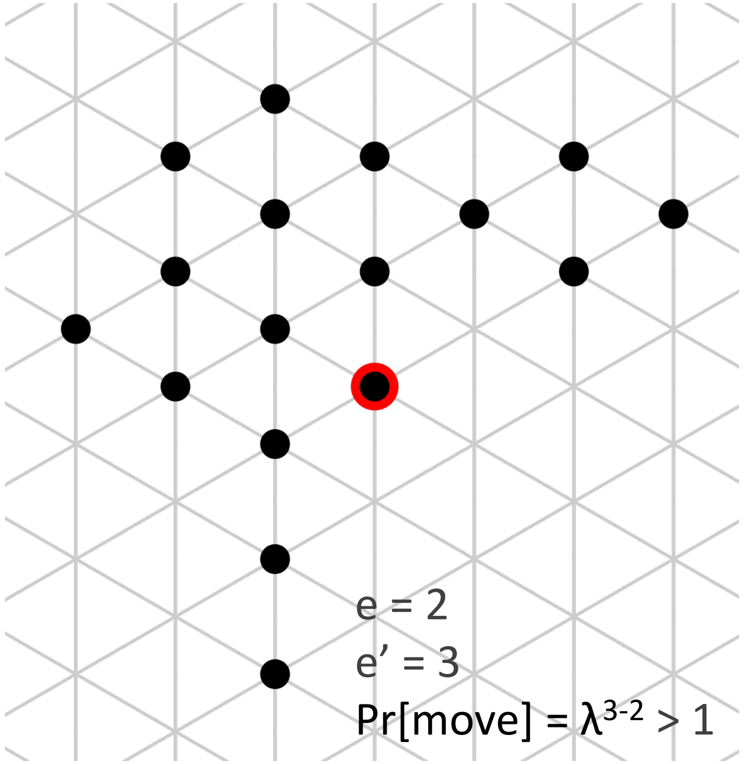

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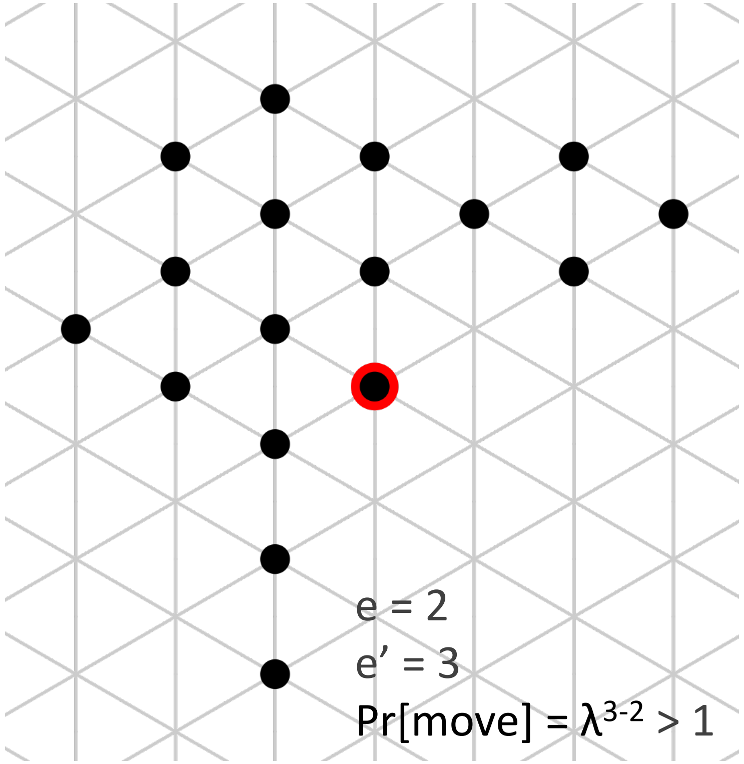

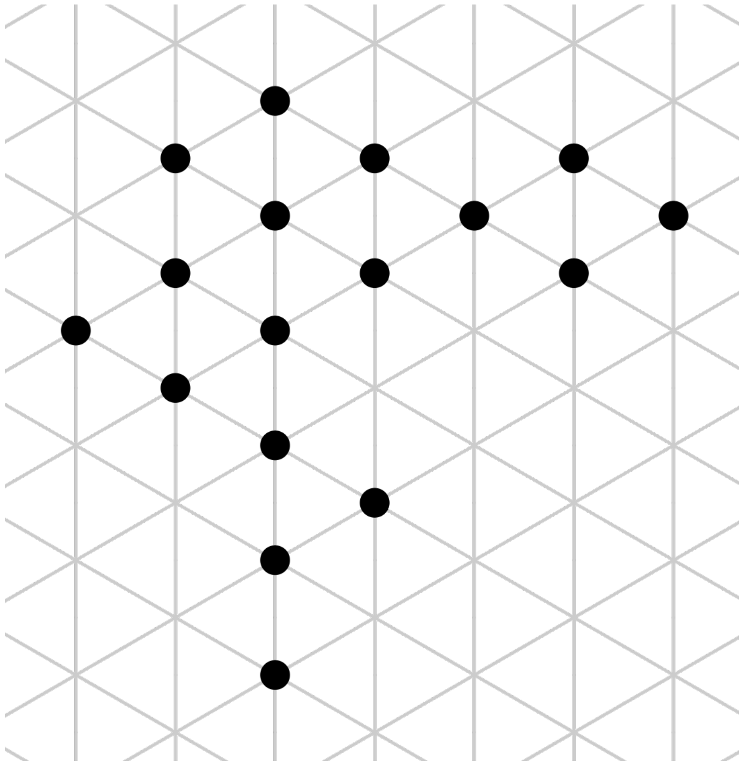

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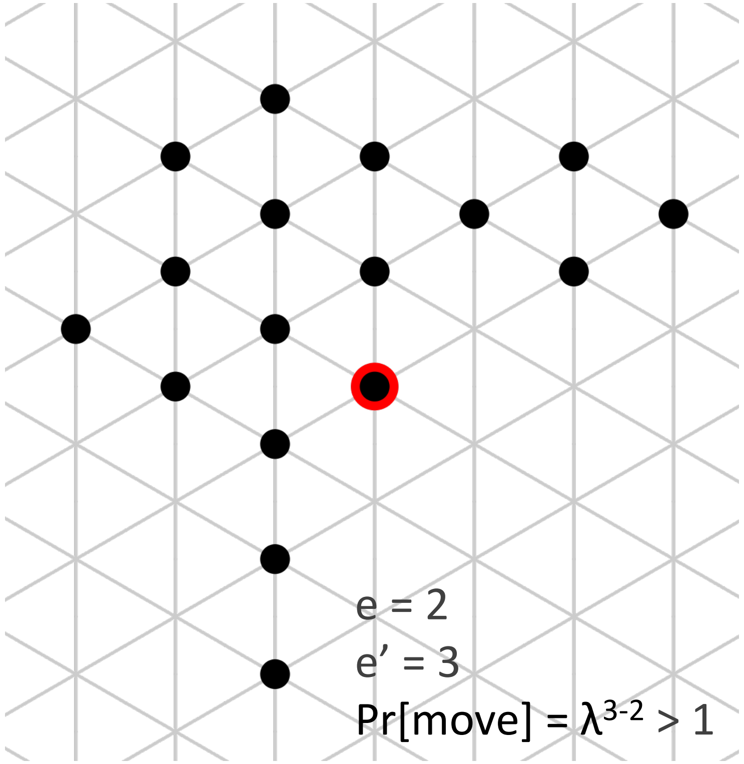

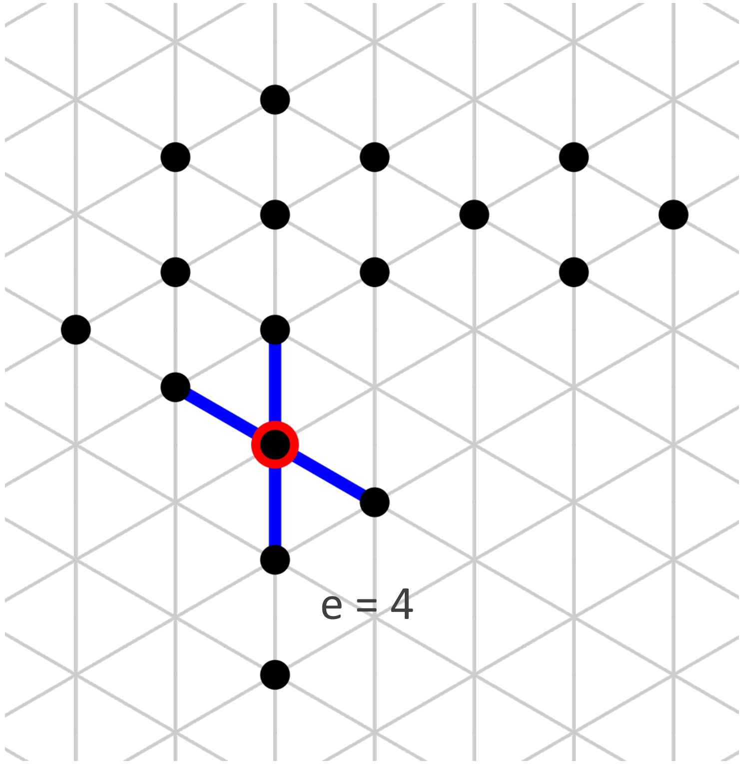

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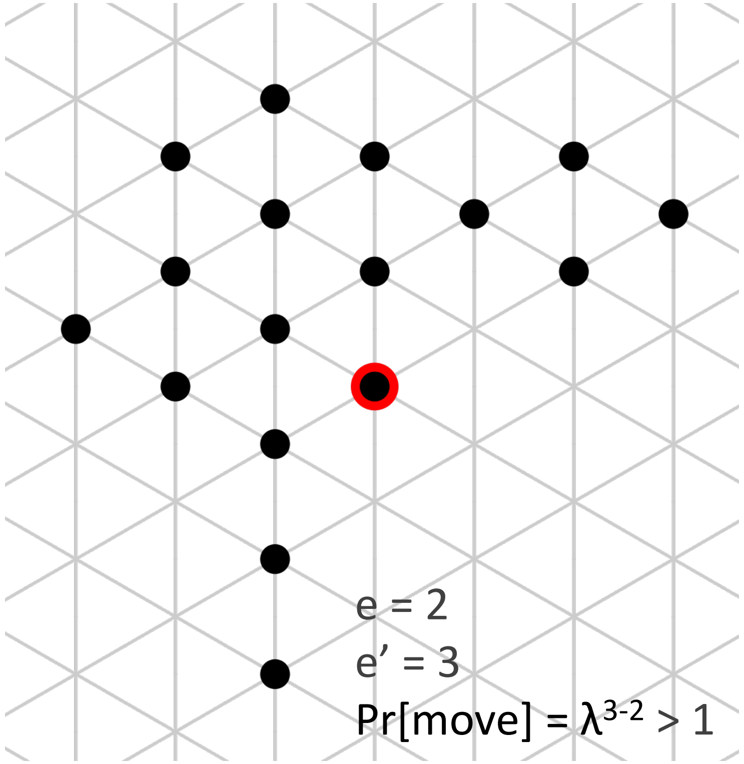

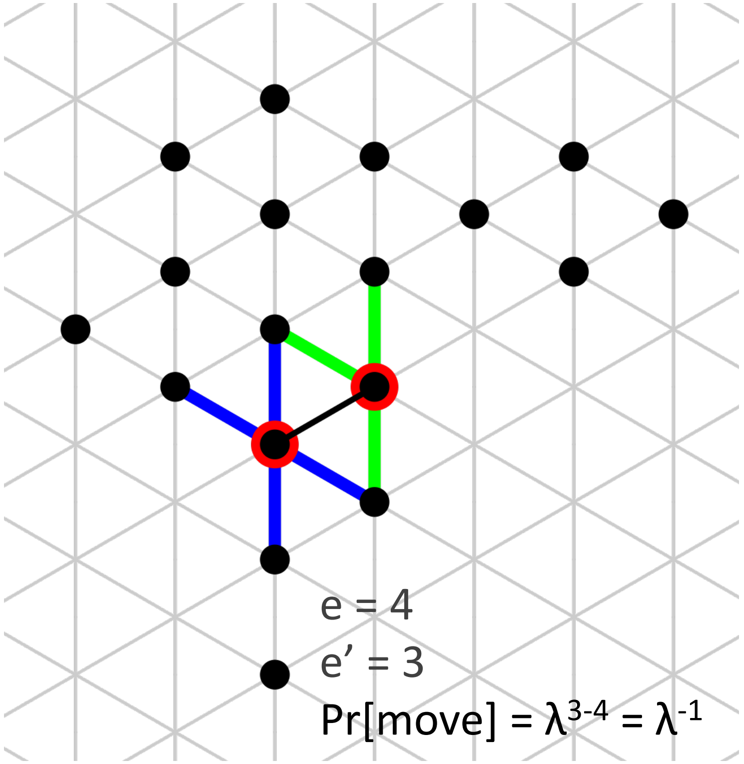

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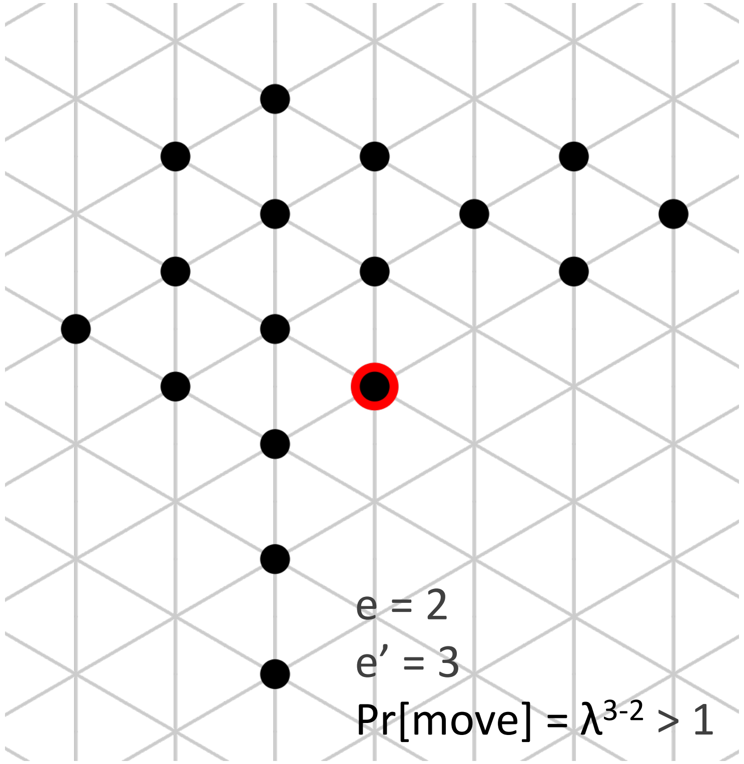
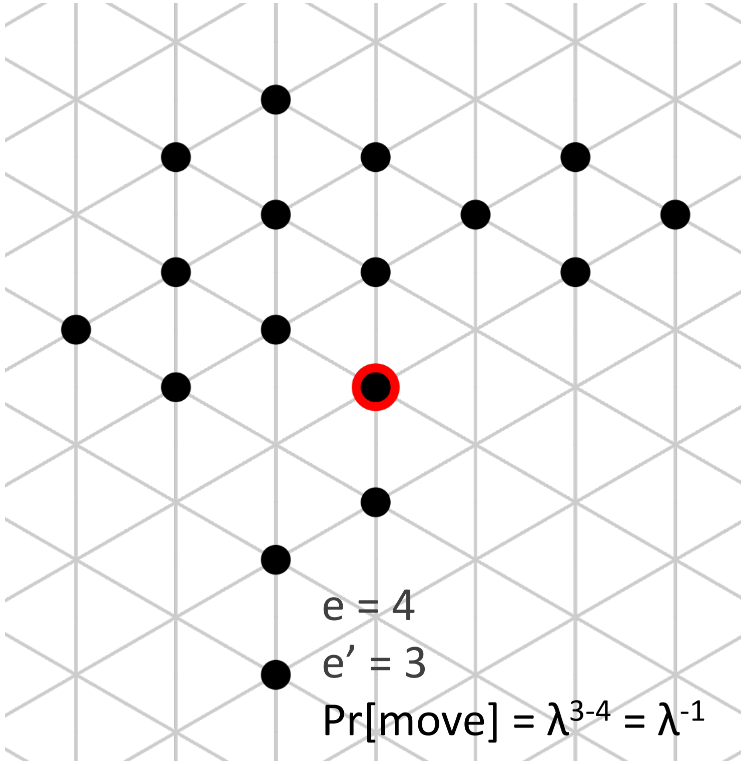
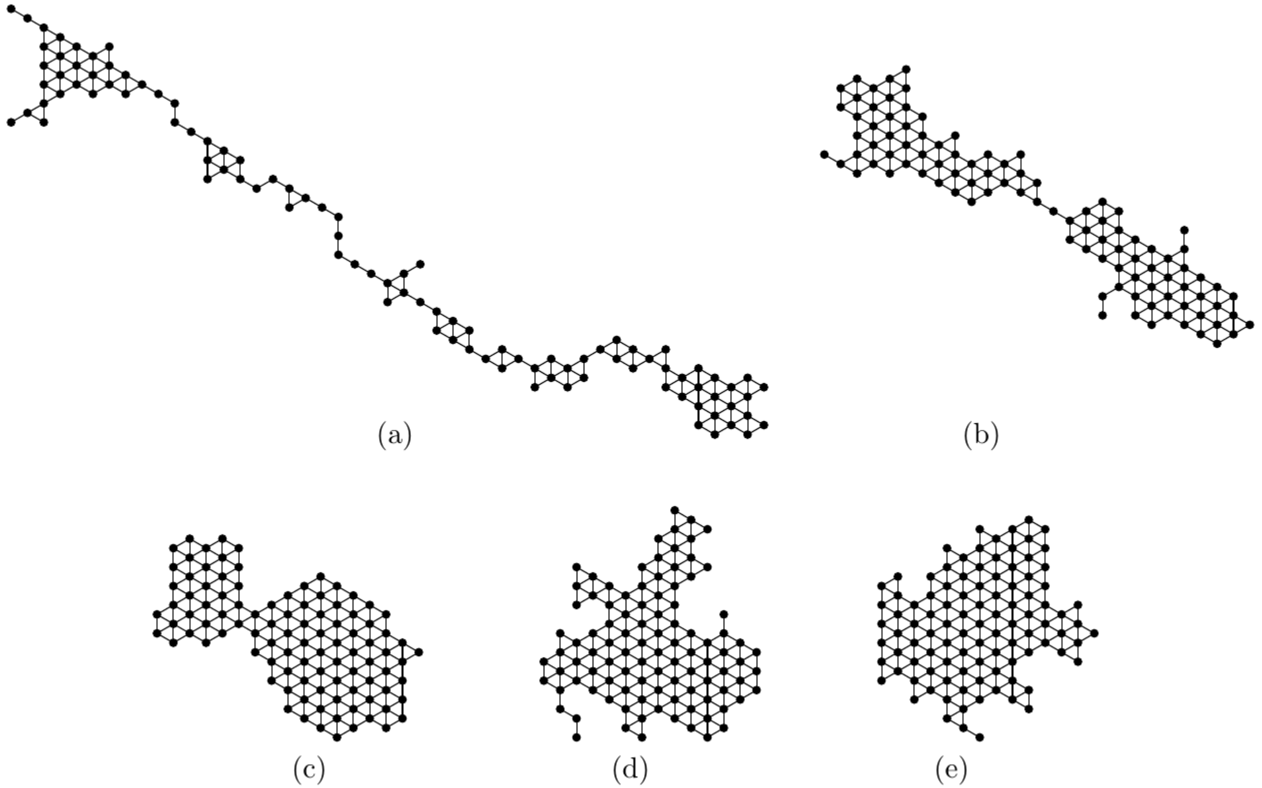




Simulation: Compression, λ = 4

100 particles initially in a line after (a) 1 million, (b) 2 million, (c) 3 million, (d) 4 million, and (e) 5 million iterations.



Theoretical Results

We can use tools from Markov chain analysis to investigate our algorithm's long-run behavior, or *stationary distribution* π.

- **Theorem:** For any $\lambda > 2 + \text{sqrt}(2)$ with $\pi(\sigma) \sim \lambda^{e(\sigma)}$, there is an $\alpha > 1$ such that, at stationarity, with all but exponentially small probability the particle system is α-compressed.
- **Theorem:** For any $\alpha > 1$, there is a λ such that for $\pi(\sigma) \sim \lambda^{e(\sigma)}$, at stationarity, with all but exponentially small probability the particle system is α-compressed.

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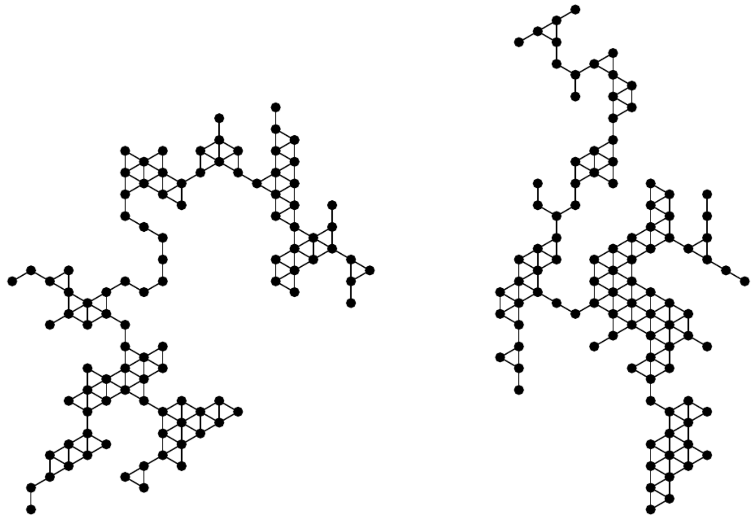
- **Theorem:** For any $\alpha > 1$, there is a λ such that for $\pi(\sigma) \sim \lambda^{e(\sigma)}$, at stationarity, with all but exponentially small probability the particle system is α-compressed.

And surprisingly…

- **Theorem:** For any $\lambda < 2.17$ with $\pi(\sigma) \sim \lambda^{e(\sigma)}$ and any $\alpha > 1$, at stationarity, the probability that the particle system is α-compressed is exponentially small.

“Expanding” Beyond Compression

The last theorem shows that we can start with a compressed system and push λ below 2.17 to get the opposite behavior: *expansion*.

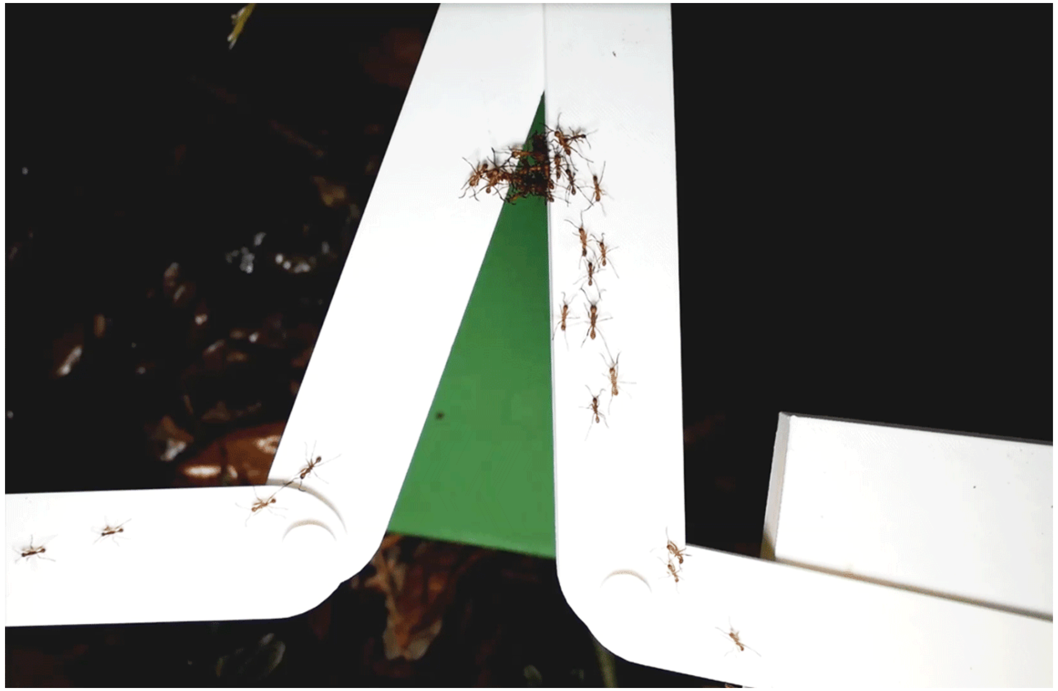


(a) (b)

What else can we do?

Shortcut Bridging: Motivation

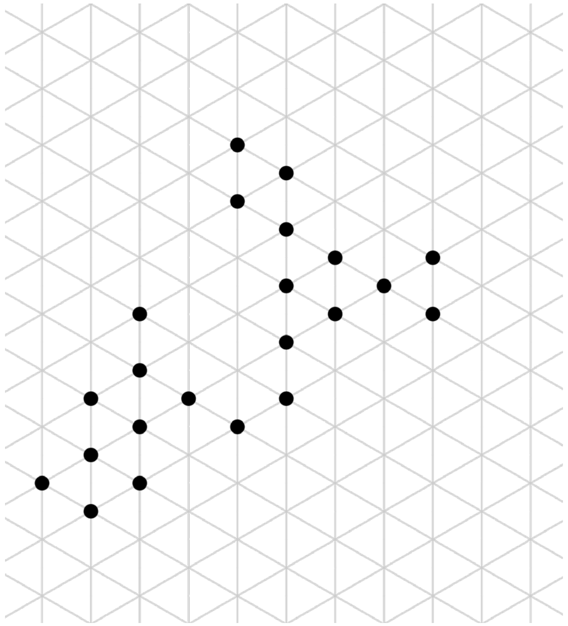
The ants in genus *Eticon* balance shortening their foraging paths with avoiding committing too many ants to the bridge, resulting in a smaller foraging force.



RLPKCG 2015: "Army ants dynamically adjust living bridges..."

Shortcut Bridging: Setting

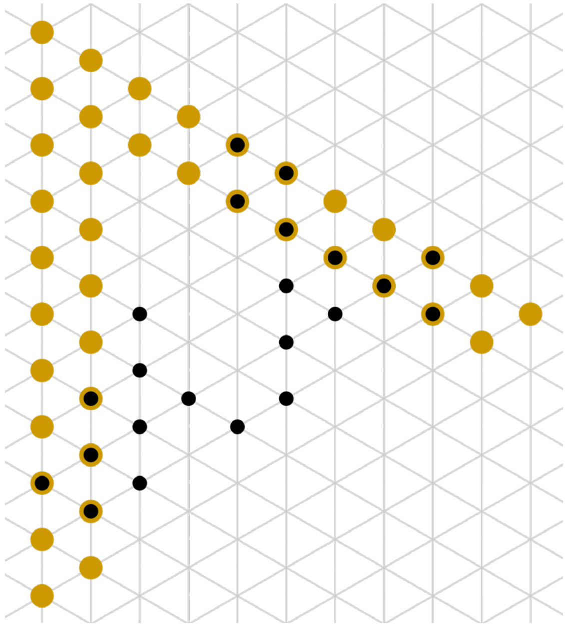
Similar setting to compression…



Shortcut Bridging: Setting

Similar setting to compression, but adding:

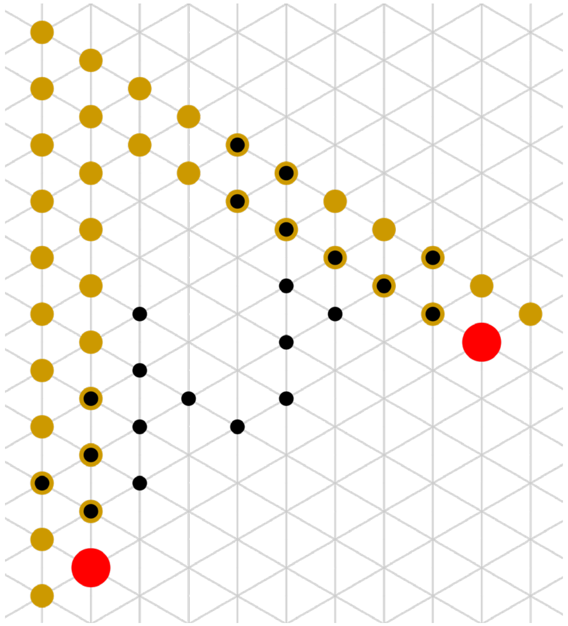
- Land & gap positions.



Shortcut Bridging: Setting

Similar setting to compression, but adding:

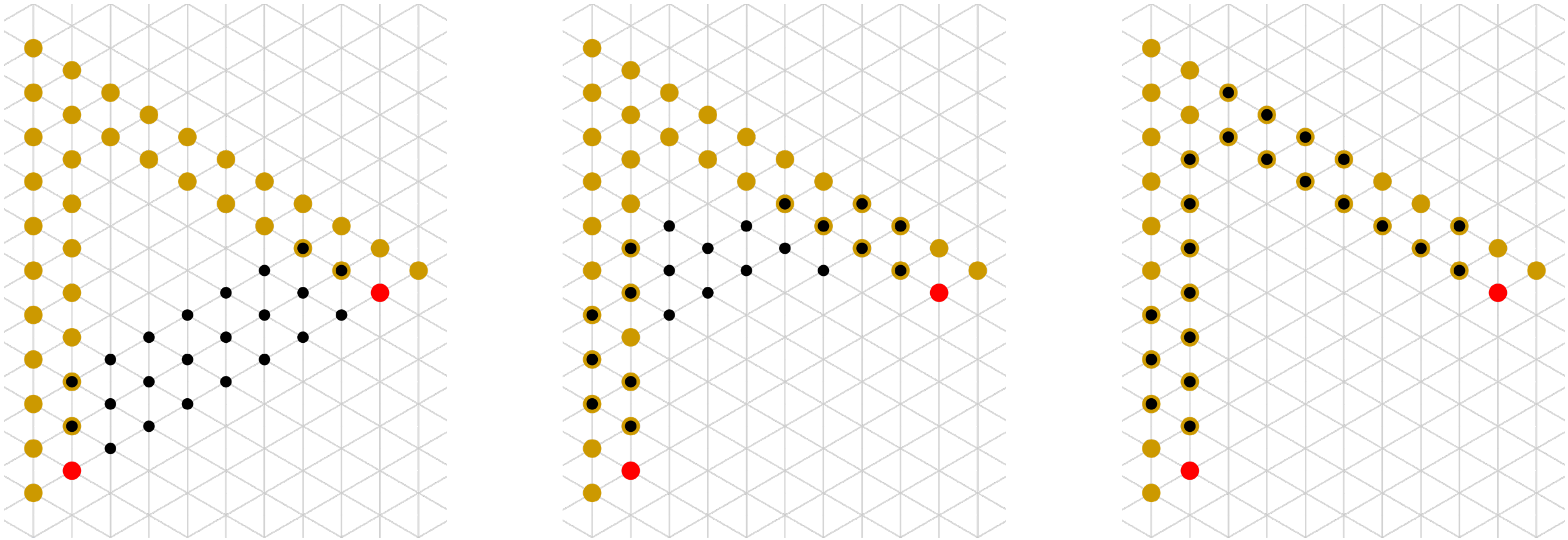
- Land & gap positions.
- Fixed objects (to anchor the particle system to land).



Shortcut Bridging: Problem Statement

Goal 2: Balance two competing objectives:

- Minimizing overall perimeter (controlled by λ, as in compression)
- Minimizing total *gap perimeter* (controlled by γ)



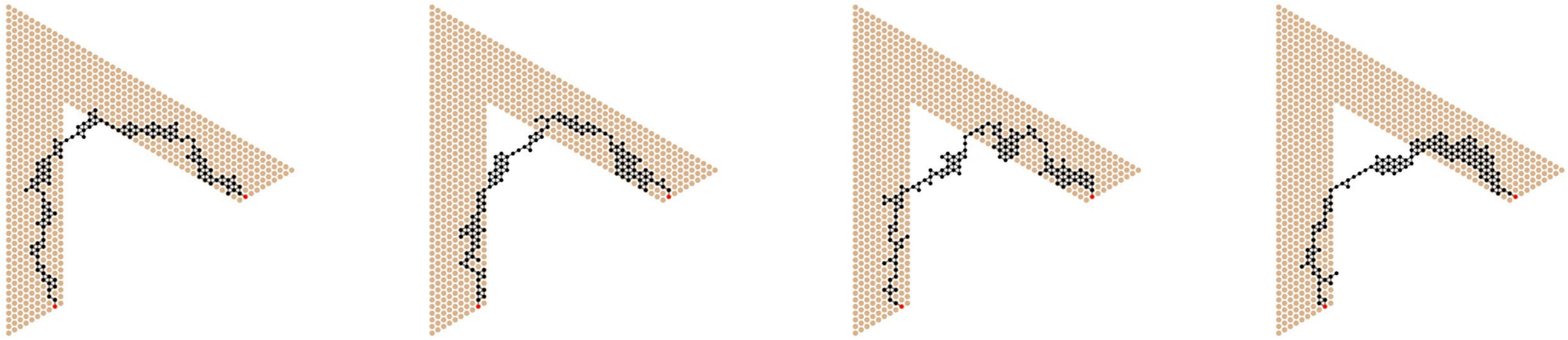
A Markov Chain for Shortcut Bridging

Input: an initial configuration σ_0 (connected, hole-free), and a bias parameters $\lambda, \gamma > 1$.

1. Choose a particle from the system uniformly at random.
2. Choose a direction from {0, … , 5} and a number p from (0,1) uniformly at random.
3. If properties for maintaining connectivity and avoiding holes hold and $p < \lambda^{\Delta p} \gamma^{\Delta g}$, then move in that direction.
4. Otherwise, do nothing.

Simulation: Shortcut Bridging, $\lambda = 4$, $\gamma = 2$

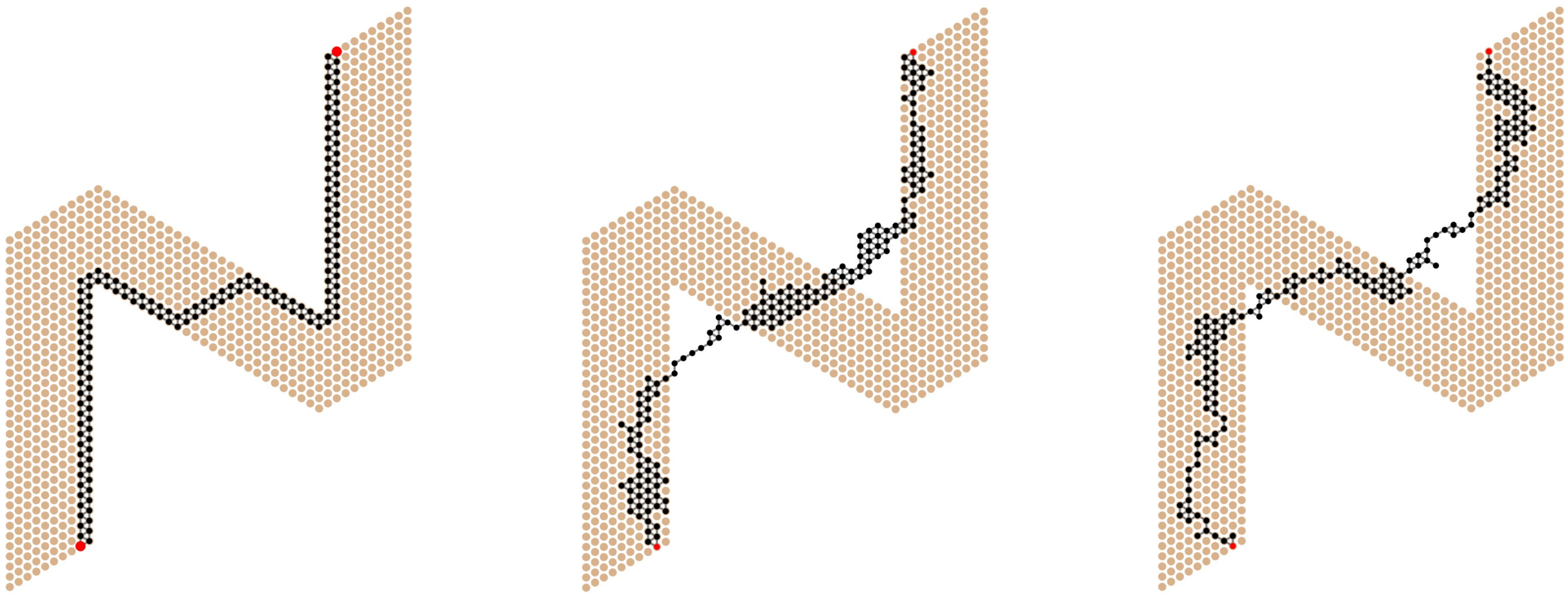
A particle system initially fully on a V-shaped land mass after (a) 2 million, (b) 4 million, (c) 6 million, and (d) 8 million iterations.



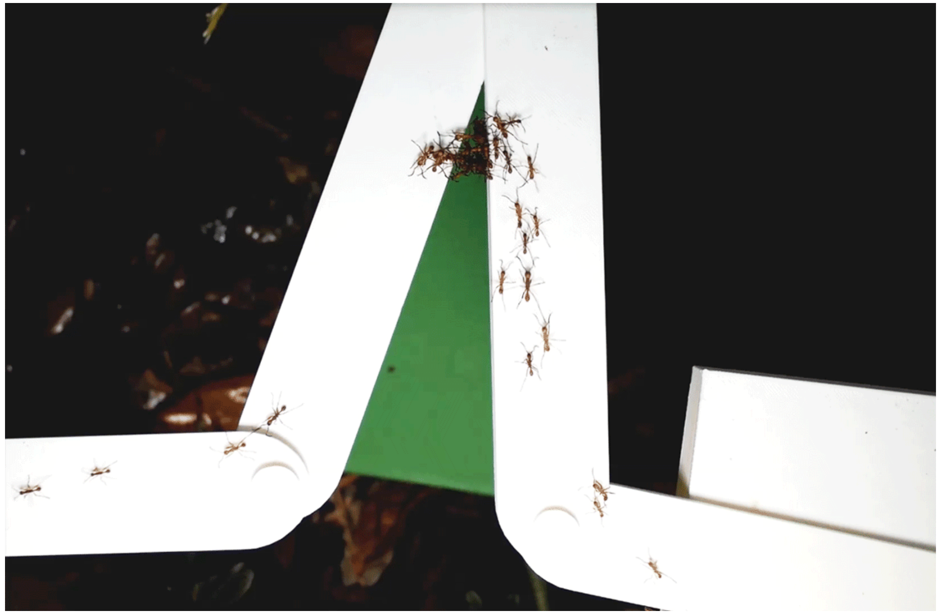
(a) (b) (c) (d)

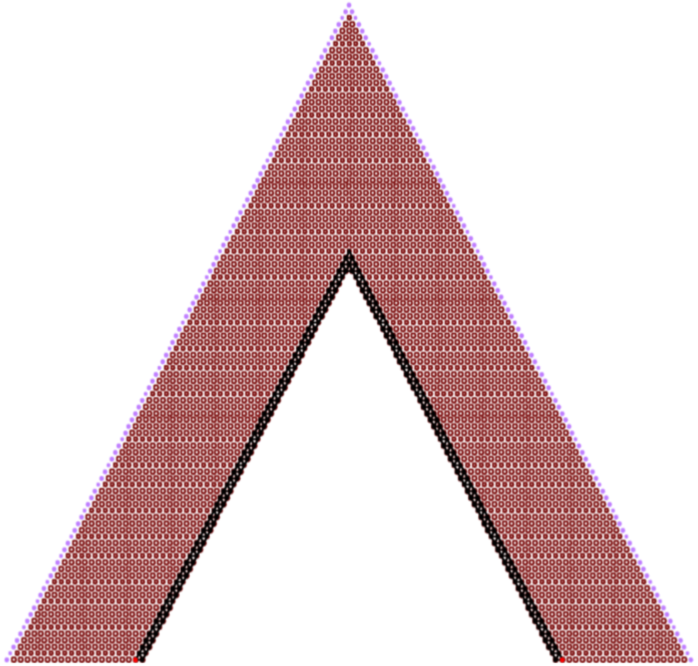
Simulation: Shortcut Bridging, $\lambda = 4$, $\gamma = 2$

A particle system initially fully on an N-shaped land mass after 10 million and 20 million iterations.



Simulation: Next to *Eticon*



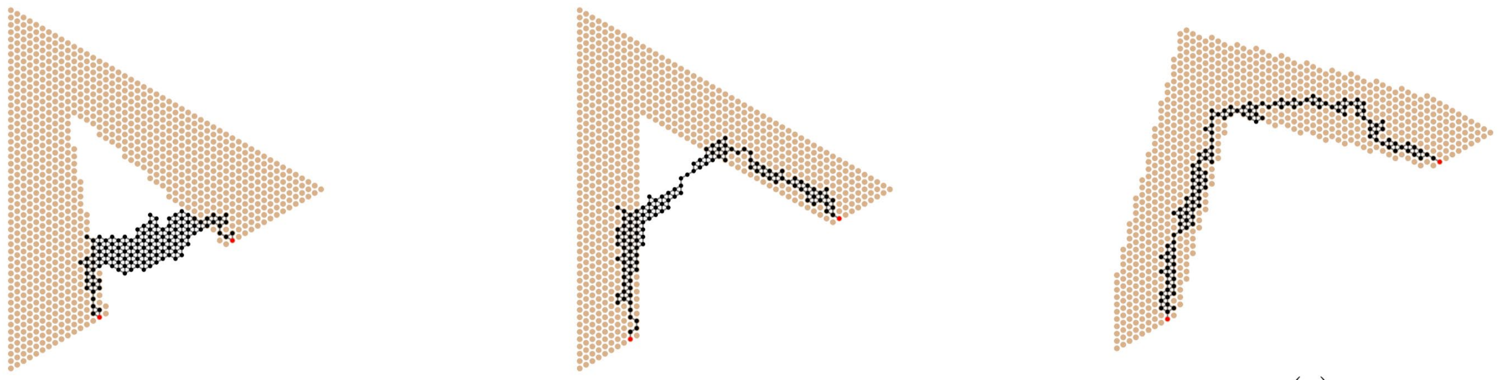


Summary of Theoretical Results

We use a metric called *weighted perimeter*, denoted $p'(\sigma)$, to capture the costs of both total and gap perimeter.

- **Theorem:** For any $\alpha > 1$, there are $\lambda > 2 + \text{sqrt}(2)$ and $\gamma > 1$ such that for $\pi(\sigma) \sim \lambda^{p(\sigma)} \gamma^{g(\sigma)}$, at stationarity, with all but exponentially small probability $p'(\sigma) \leq \alpha \cdot p'_{min}$.

Theorems on angle dependence:



(a) (b) (c)

Current & Future Work

- Compression: appeared at PODC ‘16.
- Shortcut Bridging: accepted to DNA23.
- More extensions of compression, e.g., foraging.
  - Explore systems with heterogenous bias parameters.
  - Investigate behaviors when particles can change their bias parameters over time.
  - Mix this stochastic approach with non-stochastic elements.
- “Active matter”: alignment, locomotion, and other emergent behaviors.

Collaborators

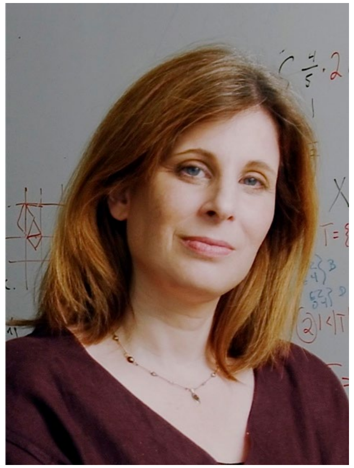
Arizona State University

Georgia Tech

UNIVERSIDAD DE GRANADA

Andréa W. Richa

Joshua J. Daymude



Dana Randall

Sarah Cannon

Marta Andrés Arroyo

Thank you!

sops.engineering.asu.edu