Algorithmic Foundations of Emergent Behavior in Analog Collectives

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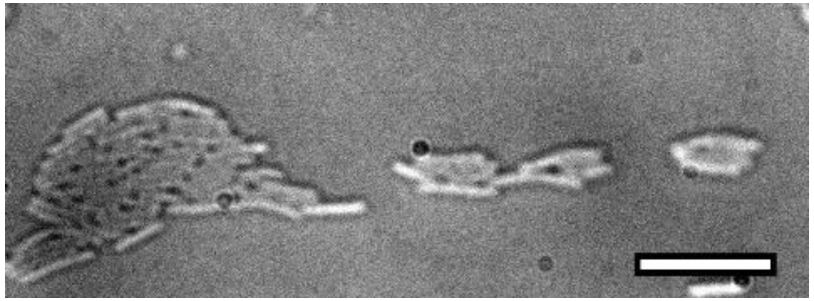
Seminar — July 14, 2021

The Santa Fe Institute

Self-Organizing Systems

Cooperative decentralized systems are capable of surprising emergent behavior arising from relatively simple interactions of their members.



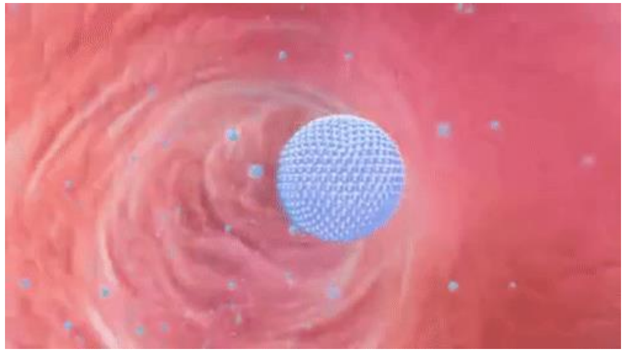
HMSKCLCA 2011

Microsoft Research 2016

Why Self-Organization?

Challenge #1: Engineering autonomous, distributed systems with arbitrary scalability.





Challenge #2: Characterizing observed emergent phenomena in complex systems.



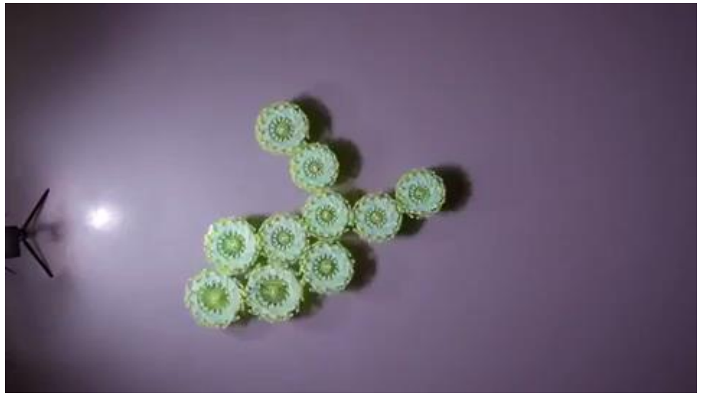
Programmable Matter

Programmable matter is a substance that can change its physical properties autonomously based on user input or environmental stimuli.

"Catoms"
PB 2018

"Kilobots"
RCN 2014

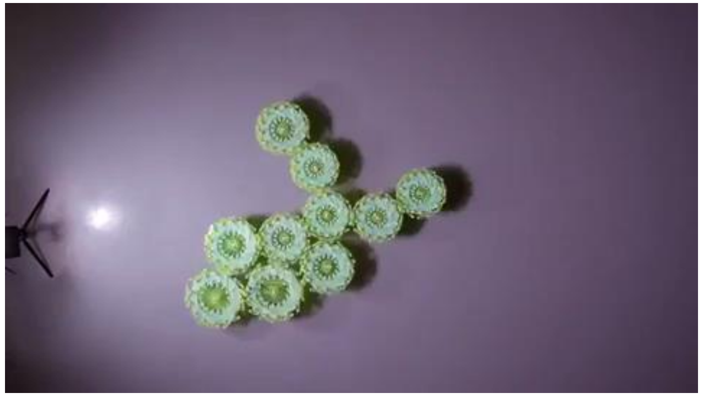
"M-Blocks"
RGR 2013

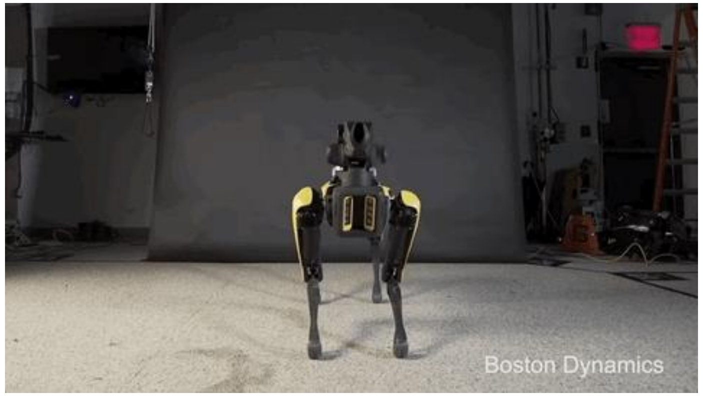


"Particle Robots"
LBBCRHRL 2019

Programmable Matter

Centimeter/millimeter-scale robots are more limited than, say, Spot from Boston Dynamics.





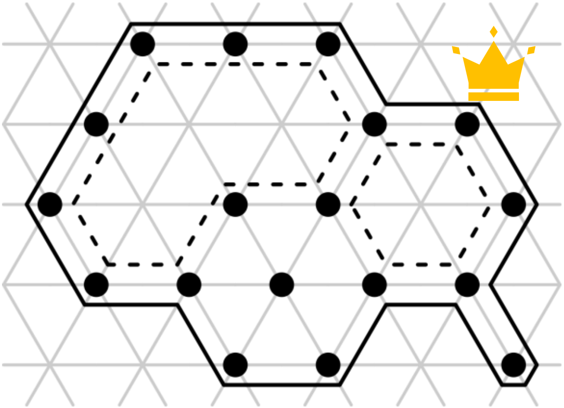
Most programmable matter and modular robotic systems assume:

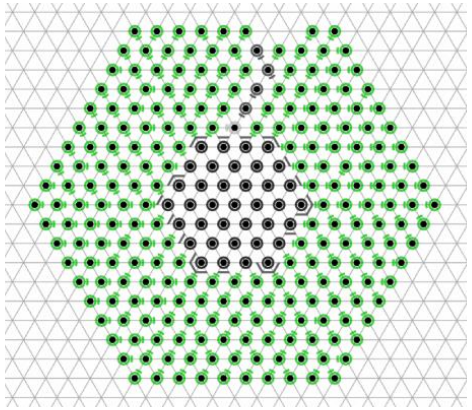
- Modest compute resources.
- Strictly local sensing and communication (e.g., 1-neighborhood).
- Limited (e.g., constant-size) or no persistent memory.
- Local, rudimentary movement.

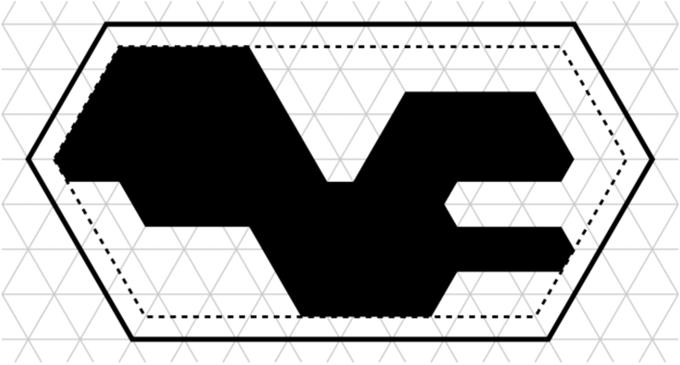
Stateful Distributed Algorithms for Programmable Matter

Under the **amoebot model** [Derakhshandeh et al. 2014] we've shown that constant-size memory, local communication between neighbors, and local movements suffice to solve:

1. **Leader Election**. A unique amoebot must irreversibly declare itself the system's leader.
2. **Object Coating**. The system must reconfigure into even layers coating a given object.
3. **Convex Hull Formation**. The system must reconfigure as the convex hull of a given object, enclosing it with the minimum number of amoebots.







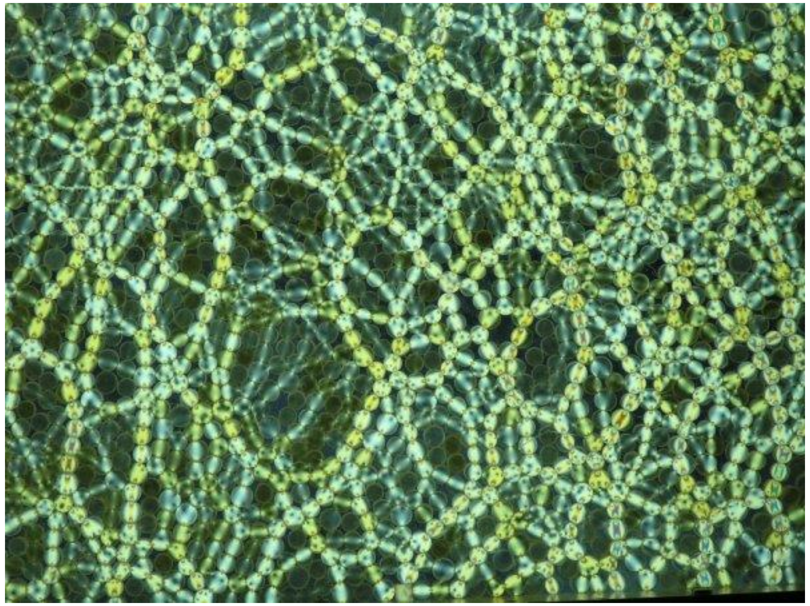
[1] **Daymude**, Gmyr, Richa, Scheideler, Strothmann. "Improved Leader Election for Self-Organizing Programmable Matter." **ALGOSENSORS** 2017.
[2] **Daymude**, Derakhshandeh, Gmyr, Porter, Richa, Scheideler, Strothmann. "On the Runtime of Universal Coating for Programmable Matter." **Natural Computing**, 2018.
[3] **Daymude**, Gmyr, Hinnenthal, Kostitsyna, Scheideler, Richa. "Convex Hull Formation for Programmable Matter." **ICDCN** 2020.

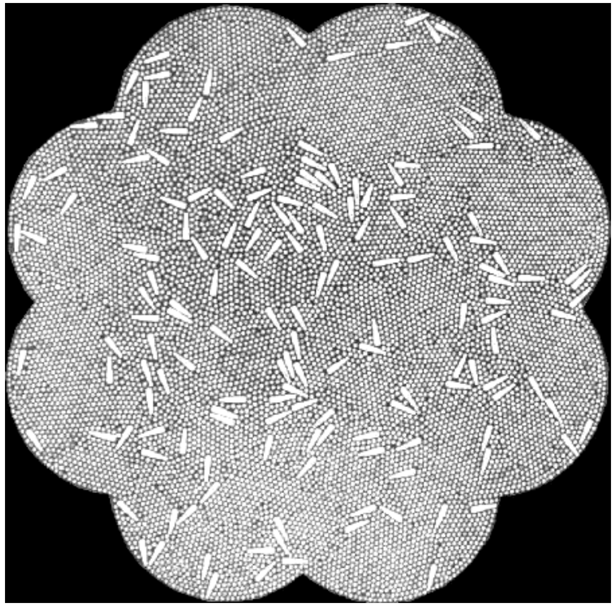
Active Granular Matter

A **granular material** is a "conglomerate of discrete, solid, macroscopic particles." [Duran 1999].

These systems don't "compute" digitally but are still capable of sophisticated collective behaviors and surprising phase changes.


Wikipedia


Bob Behringer


KSRS 2014

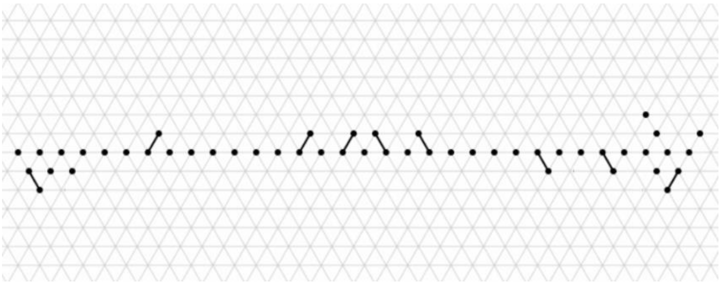
How can digital algorithms for collective behavior be translated to simple, analog systems?

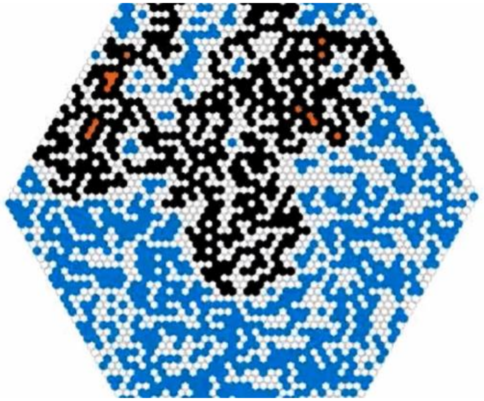
From Rigorous Algorithms to Analog Robots

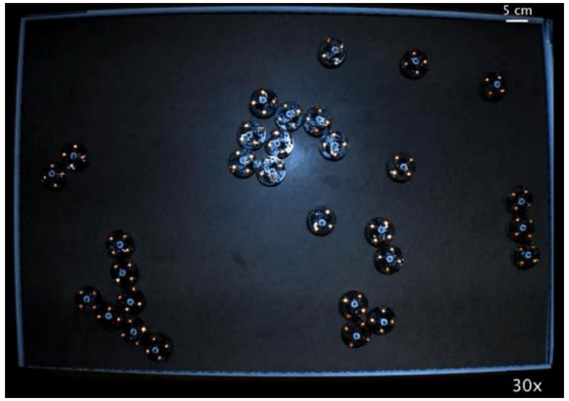
Key Idea. Leverage physical interactions to translate digital algorithms for simple analog robots.

Consider **aggregation**, where robots gather compactly, and **dispersion**, its inverse.

Markov chain $\mathcal{M}_C$ for Compression → Markov chain $\mathcal{M}_A$ for Aggregation → Aggregation in BOBbots

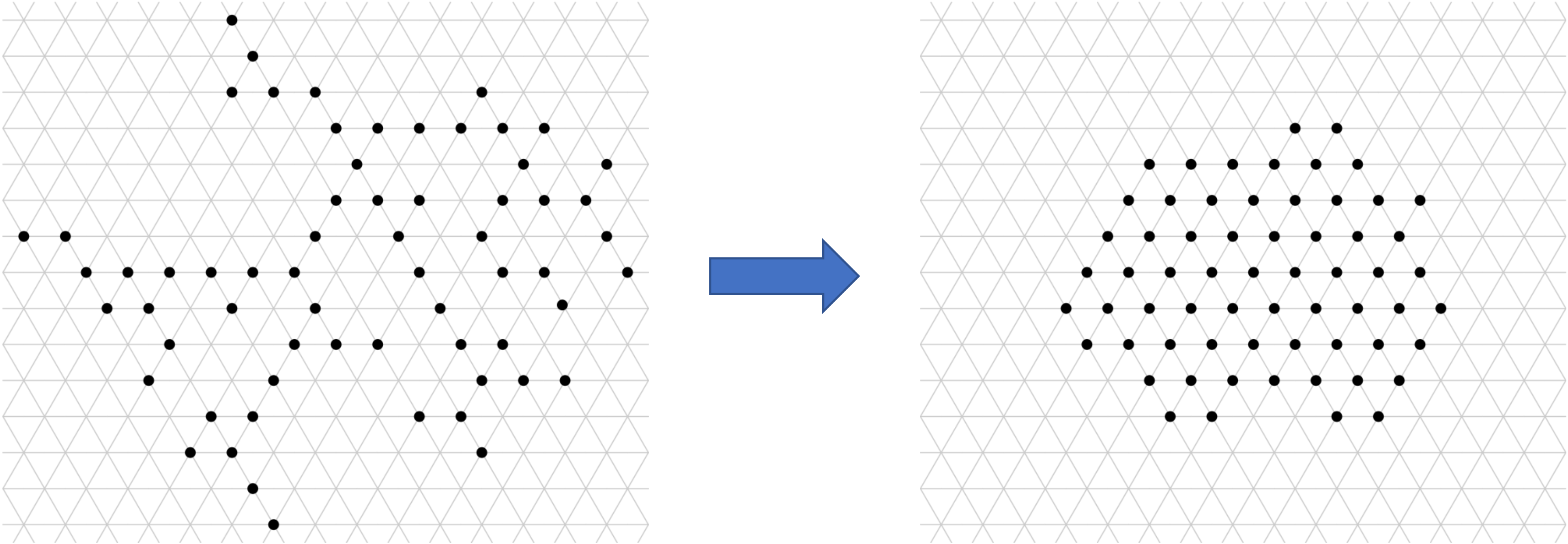






Compression

Problem. Using local, distributed rules, how can particles “compress,” gathering compactly while remaining simply connected?



Not compressed   Compressed

Definition: A configuration is ***α*-compressed** if its perimeter is at most α times the minimum perimeter (for this number of particles).

[4] Cannon, **Daymude**, Randall, Richa. “A Markov Chain Algorithm for Compression in Self-Organizing Particle Systems.” **PODC** 2016.

The Markov Chain $\mathcal{M}_C$ for Compression

This distributed, stochastic algorithm for compression:

total # of edges, or
nearest-neighbor pairs

- Ensures system connectivity on the triangular lattice.
- Uses Metropolis probabilities to converge to $\pi(\sigma) \propto \lambda^{e(\sigma)}$, for bias parameter $\lambda > 1$.

Fix $\lambda > 1$. Start in any connected configuration. Repeat:

1. Pick a random particle.
2. Pick a random neighboring node.
3. If the proposed node is empty, move with probability $\min\left\{\lambda^{e'-e}, 1\right\}$ if connectivity is maintained.
4. Otherwise, do nothing.

\# potential neighbors
− # current neighbors

[4] Cannon, **Daymude**, Randall, Richa. "A Markov Chain Algorithm for Compression in Self-Organizing Particle Systems." **PODC** 2016.

The Markov Chain $\mathcal{M}_C$ for Compression

Fix $\lambda > 1$. Start in any connected configuration. Repeat:

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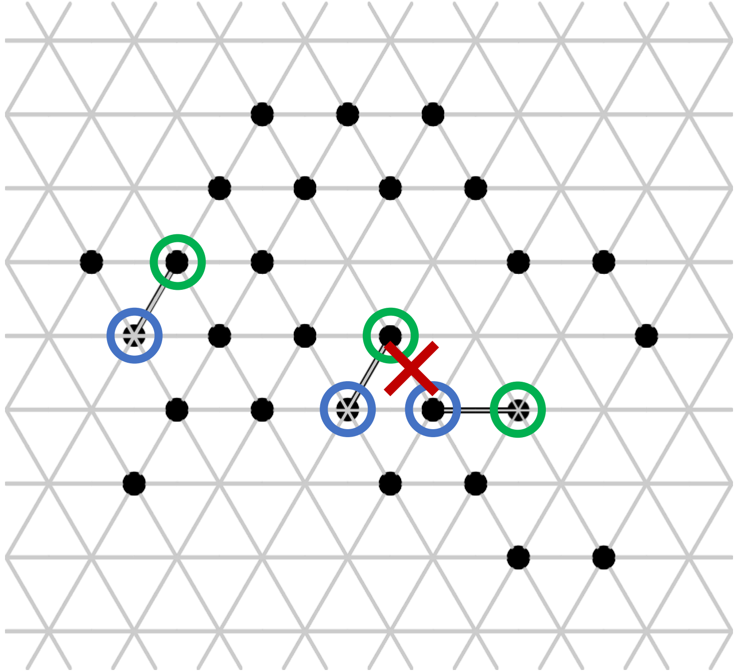
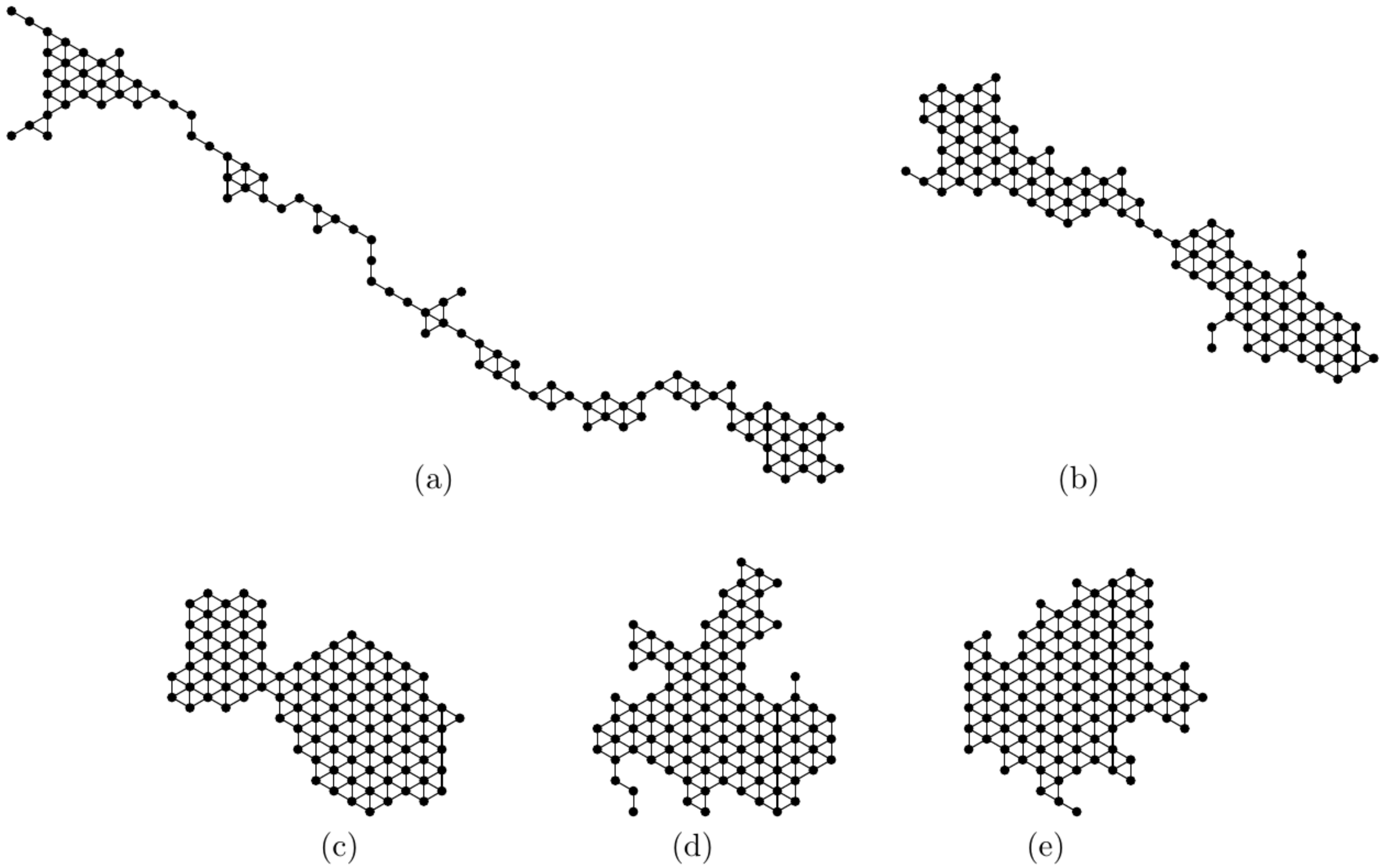


Iteration 1: $e = 3$, $e' = 4$, $\Pr[move] = \lambda^{4-3} = \lambda > 1$.

Iteration 2: $e = 4$, $e' = 2$, $\Pr[move] = \lambda^{2-4} = 1/\lambda^2$.

[4] Cannon, **Daymude**, Randall, Richa. "A Markov Chain Algorithm for Compression in Self-Organizing Particle Systems." **PODC** 2016.

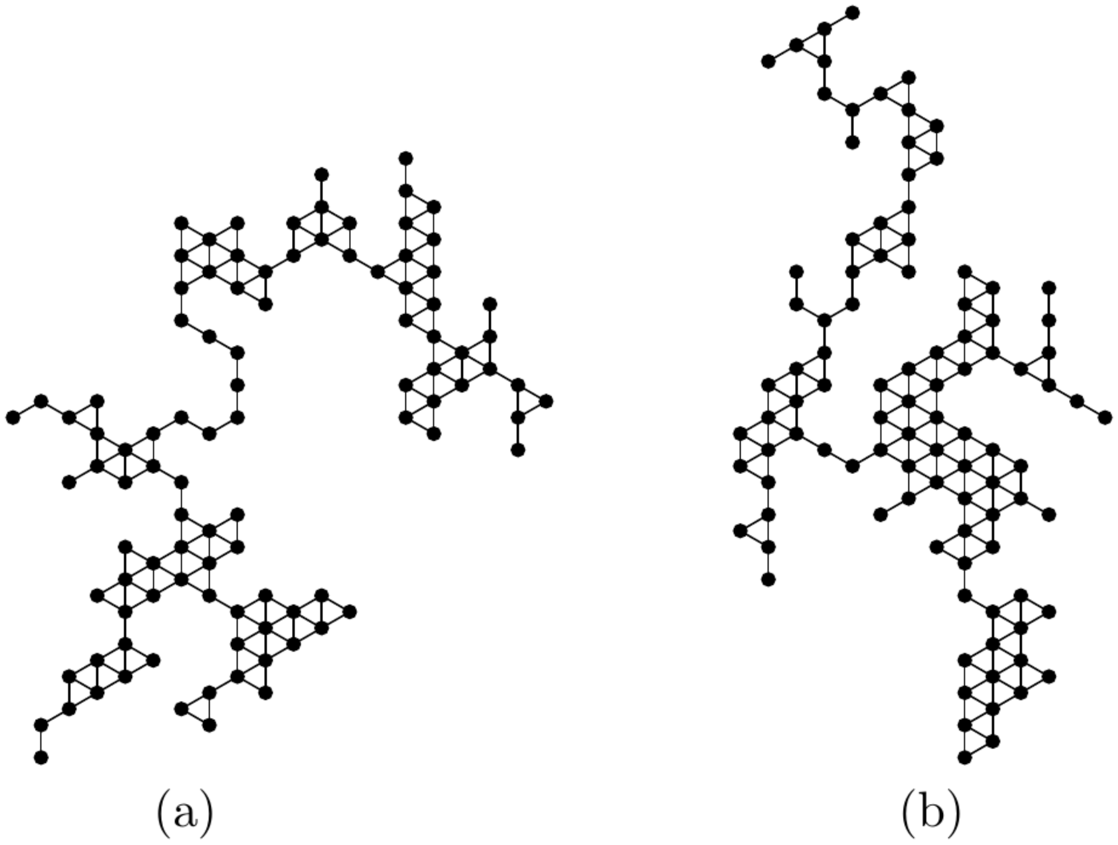
The Markov Chain $\mathcal{M}_C$ for Compression: $\lambda = 4$



100 particles after (a) 1 million, (b) 2 million, (c) 3 million, (d) 4 million, and (e) 5 million iterations, or roughly $\mathcal{O}(n^3)$ rounds.

[4] Cannon, **Daymude**, Randall, Richa. “A Markov Chain Algorithm for Compression in Self-Organizing Particle Systems.” **PODC** 2016.

The Markov Chain $\mathcal{M}_C$ for Compression: $\lambda = 2$



100 particles after (a) 10 million and (b) 20 million iterations.

[4] Cannon, **Daymude**, Randall, Richa. "A Markov Chain Algorithm for Compression in Self-Organizing Particle Systems." **PODC** 2016.

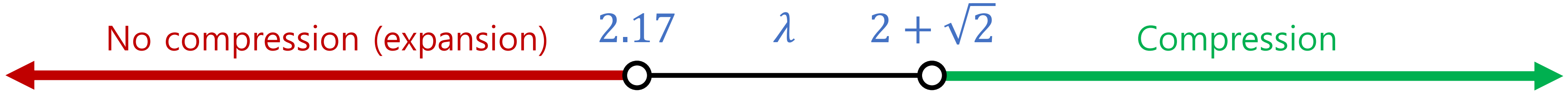
Compression: Results

Definition. A configuration is **α-compressed** if its perimeter is at most α times the minimum perimeter (for this number of particles).

Theorem. When $\lambda > 2 + \sqrt{2}$, there exists an $\alpha = \alpha(\lambda)$ such that the particle system is α-compressed at stationarity almost surely.

- For example, when $\lambda = 4$, we have $\alpha = 9$.

Theorem. When $\lambda < 2.17$, for any $\alpha > 1$, the probability the particle system is α-compressed at stationarity is exponentially small.

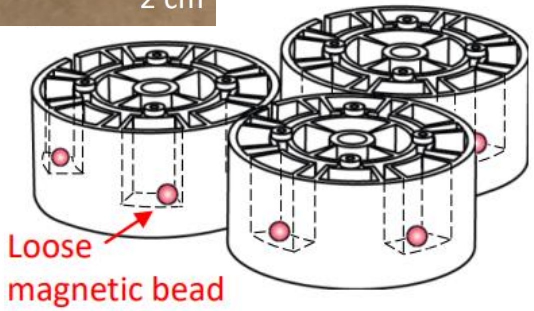


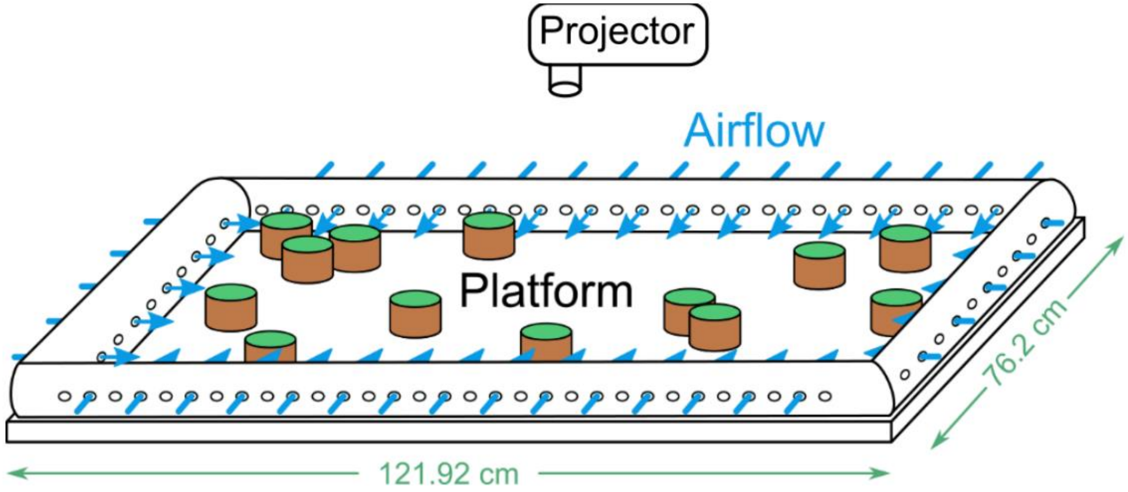
[4] Cannon, **Daymude**, Randall, Richa. “A Markov Chain Algorithm for Compression in Self-Organizing Particle Systems.” **PODC** 2016.

BOBbots: Behaving, Organizing, Buzzing Robots

Our BOBbots (named in honor of granular materials pioneer Prof. Bob Behringer, 1948-2018) replace all digital computation, sensing, and communication with physical interactions.

- Choosing a random node to move to ⇒ Noisy motion.
- Bias parameter λ ⇒ Magnets of varying strengths.





[5] Li, Dutta, Cannon, **Daymude**, Avinery, Aydin, Richa, Goldman, Randall. “Programming Active Cohesive Granular Matter with Mechanically Induced Phase Changes.” **Science Advances**, 2021.

BOBbots: Behaving, Organizing, Buzzing Robots

Many “leaps” from the theory for compression!

- Continuous vs. discrete space.
- Noisy but not random motion.
- Nonuniform robots.

[5] Li, Dutta, Cannon, **Daymude**, Avinery, Aydin, Richa, Goldman, Randall. “Programming Active Cohesive Granular Matter with Mechanically Induced Phase Changes.” **Science Advances**, 2021.

Relaxing Compression for Aggregation

Two requirements in compression that are "unnatural" for the BOBbots:

Fix $\lambda > 1$. Start in any ~~connected~~ configuration. Repeat:

1. Pick a random particle.
2. Pick a random neighboring node.
3. If the proposed node is empty, move with probability $\cancel{\min\left\{\lambda^{e'-e}, 1\right\}}$ λ^{-e} ~~if connectivity is maintained.~~
4. Otherwise, do nothing.

#1

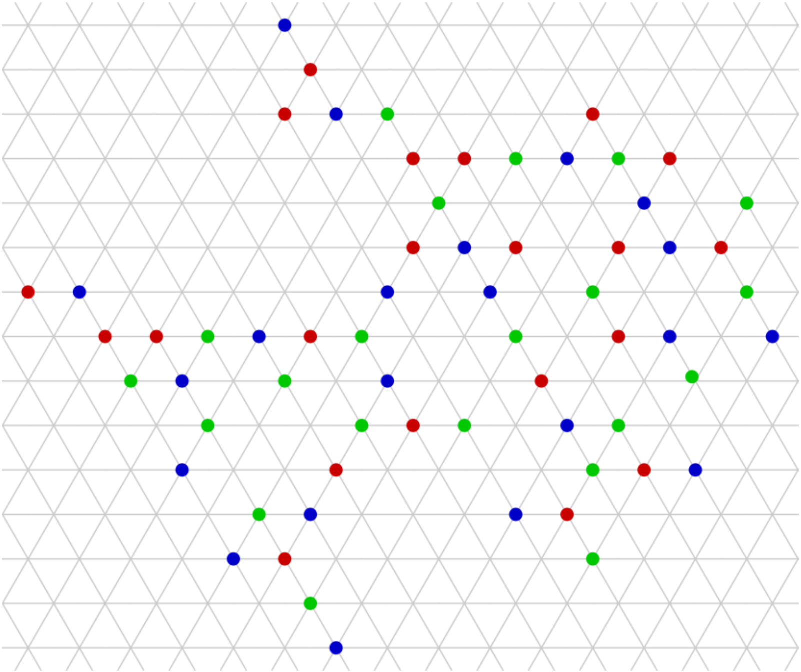
#2

1. The connectivity requirement.
2. The look ahead requirement.

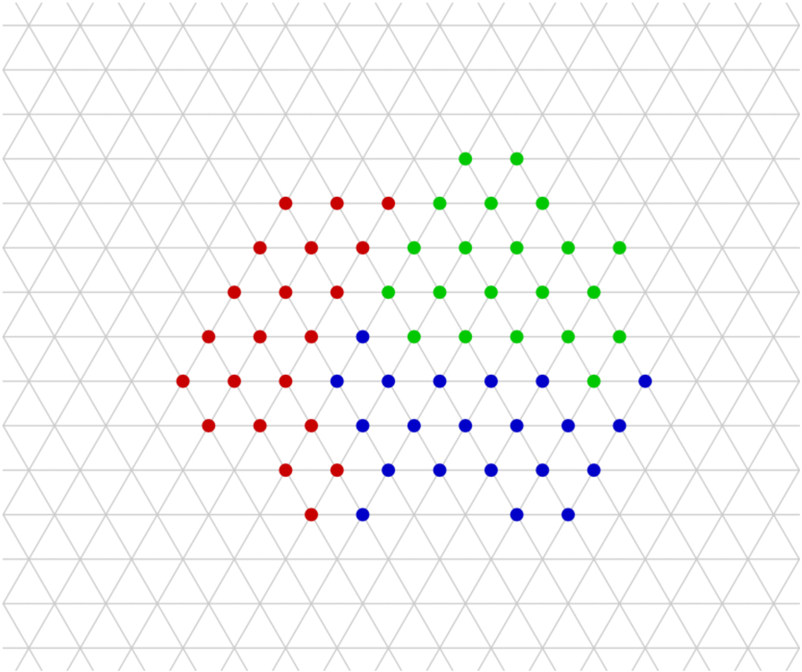
[5] Li, Dutta, Cannon, **Daymude**, Avinery, Aydin, Richa, Goldman, Randall. "Programming Active Cohesive Granular Matter with Mechanically Induced Phase Changes." **Science Advances**, 2021.

Relaxing Connectivity: Separation

Problem. Using local, distributed rules, how can heterogeneous particles "compress" overall while also separating into mostly monochromatic groups?



Neither compressed nor separated



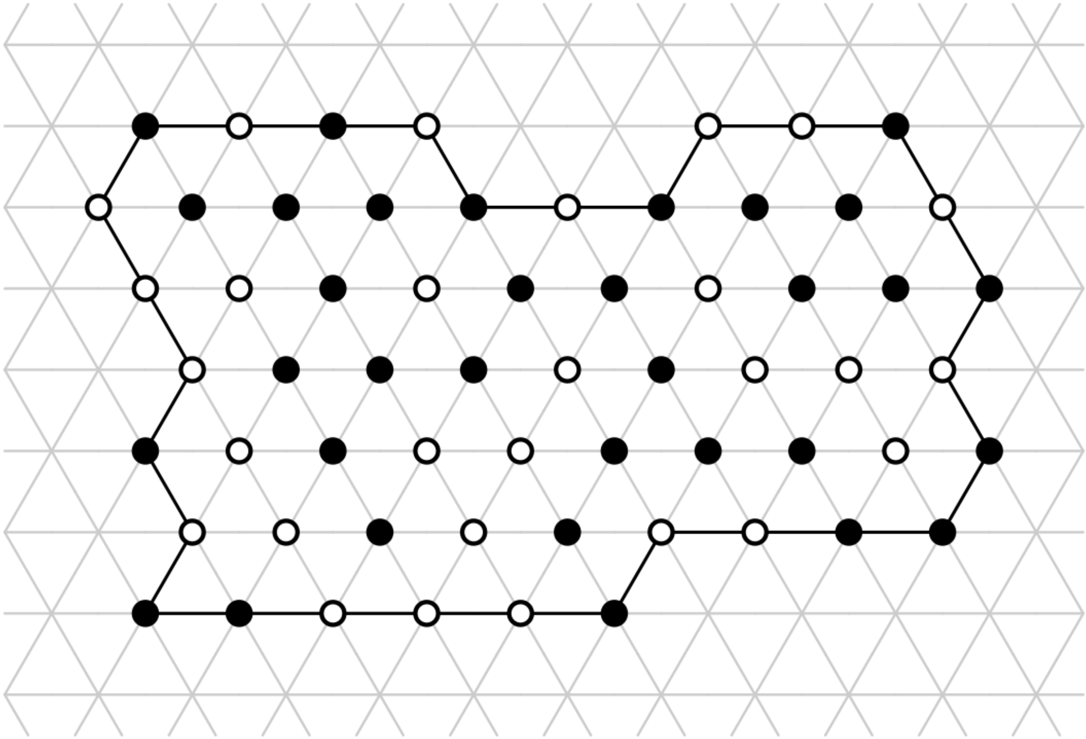
Compressed and separated

[6] Cannon, **Daymude**, Gökmen, Randall, Richa. "A Local Stochastic Algorithm for Separation in Heterogeneous Self-Organizing Particle Systems." **APPROX/RANDOM** 2019.

Relaxing Connectivity: Separation

Considered the **separation** problem in 2-color systems and proved results for:

- Particles with fixed colors that can move around.
- Particles with fixed positions (α-compressed) that can swap colors with their neighbors.



[6] Cannon, **Daymude**, Gökmen, Randall, Richa. “A Local Stochastic Algorithm for Separation in Heterogeneous Self-Organizing Particle Systems.” **APPROX/RANDOM** 2019.

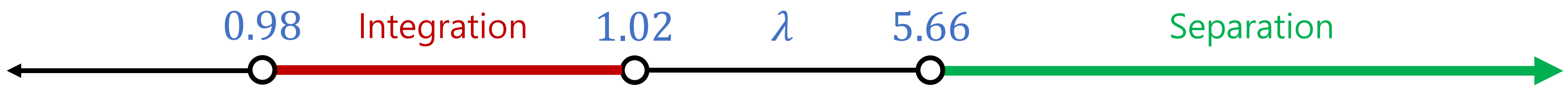
Relaxing Connectivity: Separation

Definition. A configuration is $(\boldsymbol{\beta}, \boldsymbol{\delta})$**-separated** if there is a subset R of particles such that:

1. At most $\beta\sqrt{n}$ edges have exactly one endpoint in R.
2. The density of particles of color c_1 in R is at least $1 - \delta$.
3. The density of particles of color c_1 not in R is at most δ.

A configuration is **integrated** if no such (β, δ) exist.

Theorem. Among bicolored α-compressed configurations, when $\lambda > 5.66$, there exist β, δ such that the particle system is (β, δ)-separated almost surely. However, when $0.98 < \lambda < 1.02$, the particle system is integrated almost surely.



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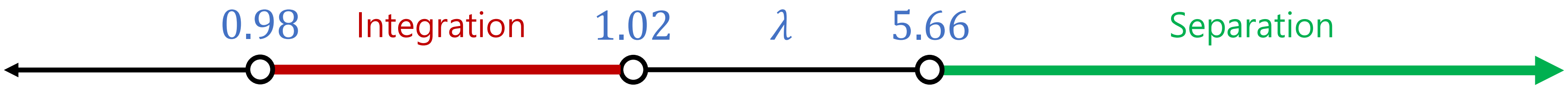
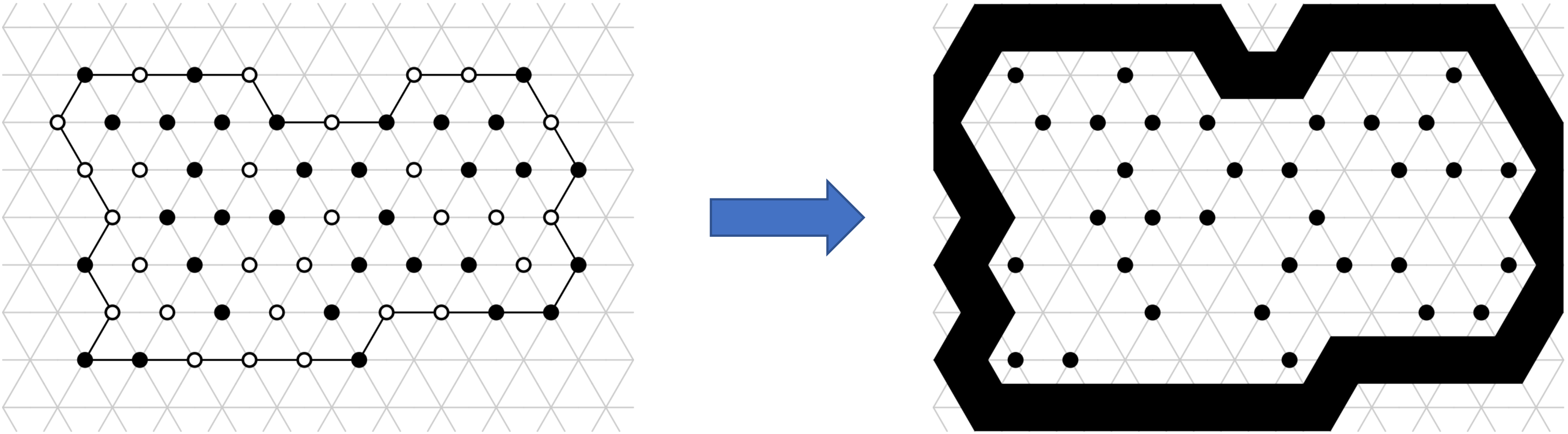
Relaxing Connectivity: Separation



This phase change provably occurs among α-compressed configurations, even in the case where particles have fixed positions but swap colors with their neighbors.

What if we treat black particles as real and white particles as unoccupied nodes?



Key Idea. Our results for separation also hold for aggregation in the disconnected setting!

[6] Cannon, **Daymude**, Gökmen, Randall, Richa. "A Local Stochastic Algorithm for Separation in Heterogeneous Self-Organizing Particle Systems." **APPROX/RANDOM** 2019.

Relaxing Compression for Aggregation

Two requirements in compression that are "unnatural" for the BOBbots:

Fix $\lambda > 1$. Start in any ~~connected~~ configuration. Repeat:

1. Pick a random particle.
2. Pick a random neighboring node.
3. If the proposed node is empty, move with probability $\min\left\{\lambda^{e'-e}, 1\right\}$ ~~if connectivity is maintained.~~
4. Otherwise, do nothing.

#2

#1

1. ~~The connectivity requirement.~~ We use separation to generalize to the disconnected setting.
2. The look ahead requirement.

[5] Li, Dutta, Cannon, **Daymude**, Avinery, Aydin, Richa, Goldman, Randall. "Programming Active Cohesive Granular Matter with Mechanically Induced Phase Changes." **Science Advances**, 2021.

Relaxing Look Ahead

The $\min\left\{\lambda^{e'-e}, 1\right\}$ transition probabilities come from the Metropolis-Hastings algorithm.

In Metropolis-Hastings, if we want a Markov chain to converge to a stationary distribution π, then we set the transition probability $P(\sigma, \tau) = \min\left\{\frac{\pi(\tau)}{\pi(\sigma)}, 1\right\}$.

Want to converge to $\pi(\sigma) \propto \lambda^{e(\sigma)}$, where $e(\sigma)$ is the number of neighboring pairs in σ.

So, when $\sigma \to \tau$ is the movement of a single particle p, we have:

$$P(\sigma, \tau) = \min\left\{\frac{\pi(\tau)}{\pi(\sigma)}, 1\right\} = \min\left\{\frac{\lambda^{e(\tau)}}{\lambda^{e(\sigma)}}, 1\right\} = \min\left\{\lambda^{e(\tau)-e(\sigma)}, 1\right\} = \min\left\{\lambda^{e'-e}, 1\right\}$$

where p has e neighbors in σ and e' neighbors in τ.

[5] Li, Dutta, Cannon, **Daymude**, Avinery, Aydin, Richa, Goldman, Randall. "Programming Active Cohesive Granular Matter with Mechanically Induced Phase Changes." **Science Advances**, 2021.

Relaxing Look Ahead

Claim. Transition probabilities λ^{-e} also converge to $\pi(\sigma) \propto \lambda^{e(\sigma)}$ but only use current neighborhood information!

Verify this by checking **detailed balance**: π is the unique stationary distribution of a Markov chain with transition probabilities P if:

$$\pi(\sigma)P(\sigma,\tau) = \pi(\tau)P(\tau,\sigma) \text{ for all } \sigma,\tau$$

<u>Proof.</u> Consider any $\sigma \to \tau$ differing by the move of a single particle p with e neighbors in σ and e' neighbors in τ. Then:

$$\frac{P(\sigma,\tau)}{P(\tau,\sigma)} = \frac{\lambda^{-e}}{\lambda^{-e'}} = \lambda^{e'-e} = \frac{\pi(\tau)}{\pi(\sigma)}$$

where the final equality follows from Metropolis-Hastings (last slide).

Rearranging terms recovers detailed balance. ■

[5] Li, Dutta, Cannon, **Daymude**, Avinery, Aydin, Richa, Goldman, Randall. "Programming Active Cohesive Granular Matter with Mechanically Induced Phase Changes." **Science Advances**, 2021.

Relaxing Compression for Aggregation

Two requirements in compression that are "unnatural" for the BOBbots:

Fix $\lambda > 1$. Start in any ~~connected~~ configuration. Repeat:

1. Pick a random particle.
2. Pick a random neighboring node.
3. If the proposed node is empty, move with probability $\cancel{\min\left\{\lambda^{e'-e}, 1\right\}}$ ~~if connectivity is maintained.~~

   λ^{-e}
4. Otherwise, do nothing.

#1

#2

1. ~~The connectivity requirement.~~ We use separation to generalize to the disconnected setting.
2. ~~The look ahead requirement.~~ Transition probabilities λ^{-e} still converge to $\pi(\sigma) \propto \lambda^{e(\sigma)}$.

[5] Li, Dutta, Cannon, **Daymude**, Avinery, Aydin, Richa, Goldman, Randall. "Programming Active Cohesive Granular Matter with Mechanically Induced Phase Changes." **Science Advances**, 2021.

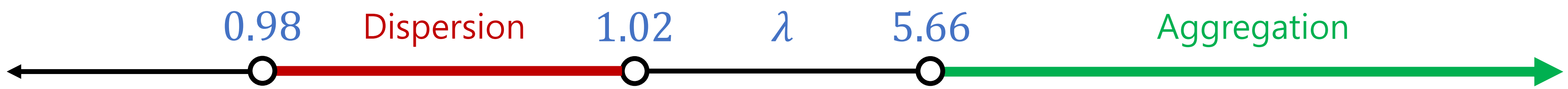
The Markov Chain $\mathcal{M}_A$ for Aggregation

Fix $\lambda > 1$. Start in any (bounded) configuration. Repeat:

1. Pick a random particle.
2. Pick a random neighboring node.
3. If the proposed node is empty, move with probability λ^{-e}.
4. Otherwise, do nothing.

Lemma. The unique stationary distribution of $\mathcal{M}_A$ is $\pi(\sigma) \propto \lambda^{e(\sigma)}$.

Theorem. Among configurations with α-compressed boundaries, when $\lambda > 5.66$, there exist β, δ such that the particle system is (β, δ)-aggregated almost surely. However, when $0.98 < \lambda < 1.02$, the particle system is dispersed almost surely.



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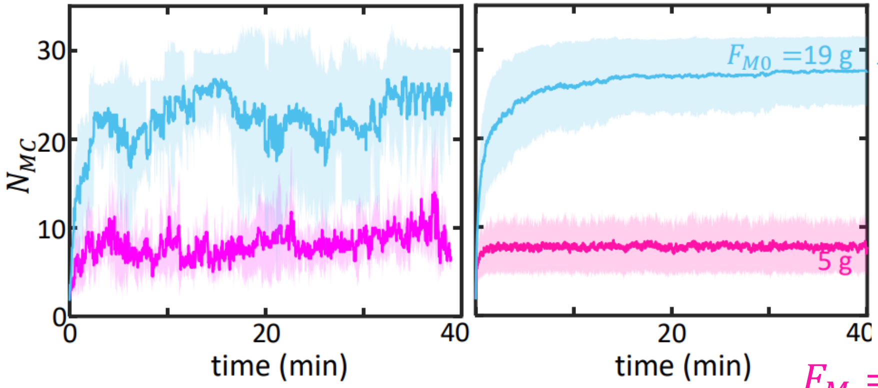
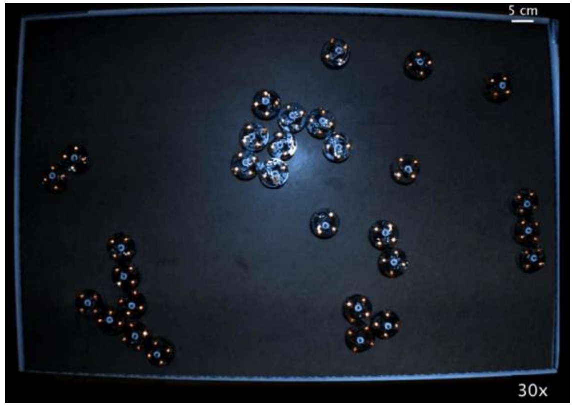
BOBbots: Aggregation and Dispersion

Our proofs indicate a phase change from dispersion to aggregation in λ-space.

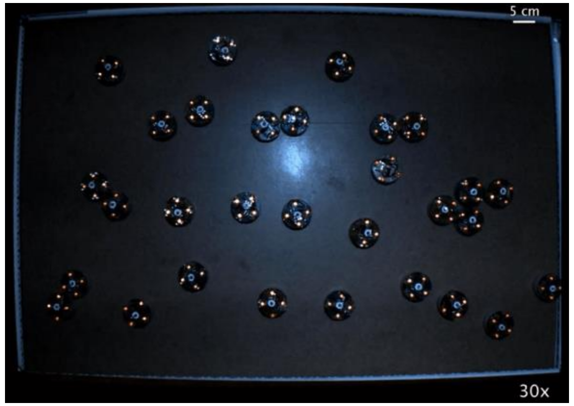
BOBbot experiments indicate the same in magnet strength (F_M) space.

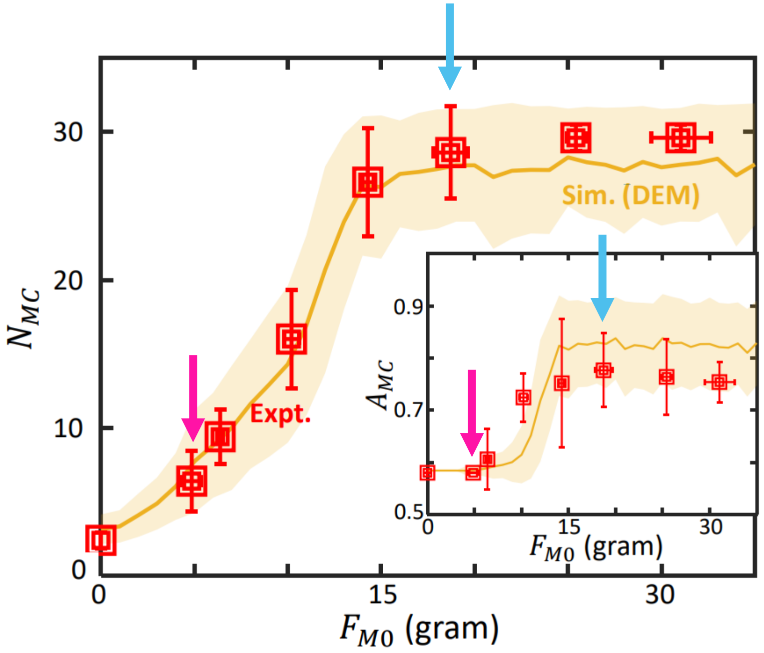


$F_M = 19$g
(Aggregation)



$F_M = 5$g
(Dispersion)





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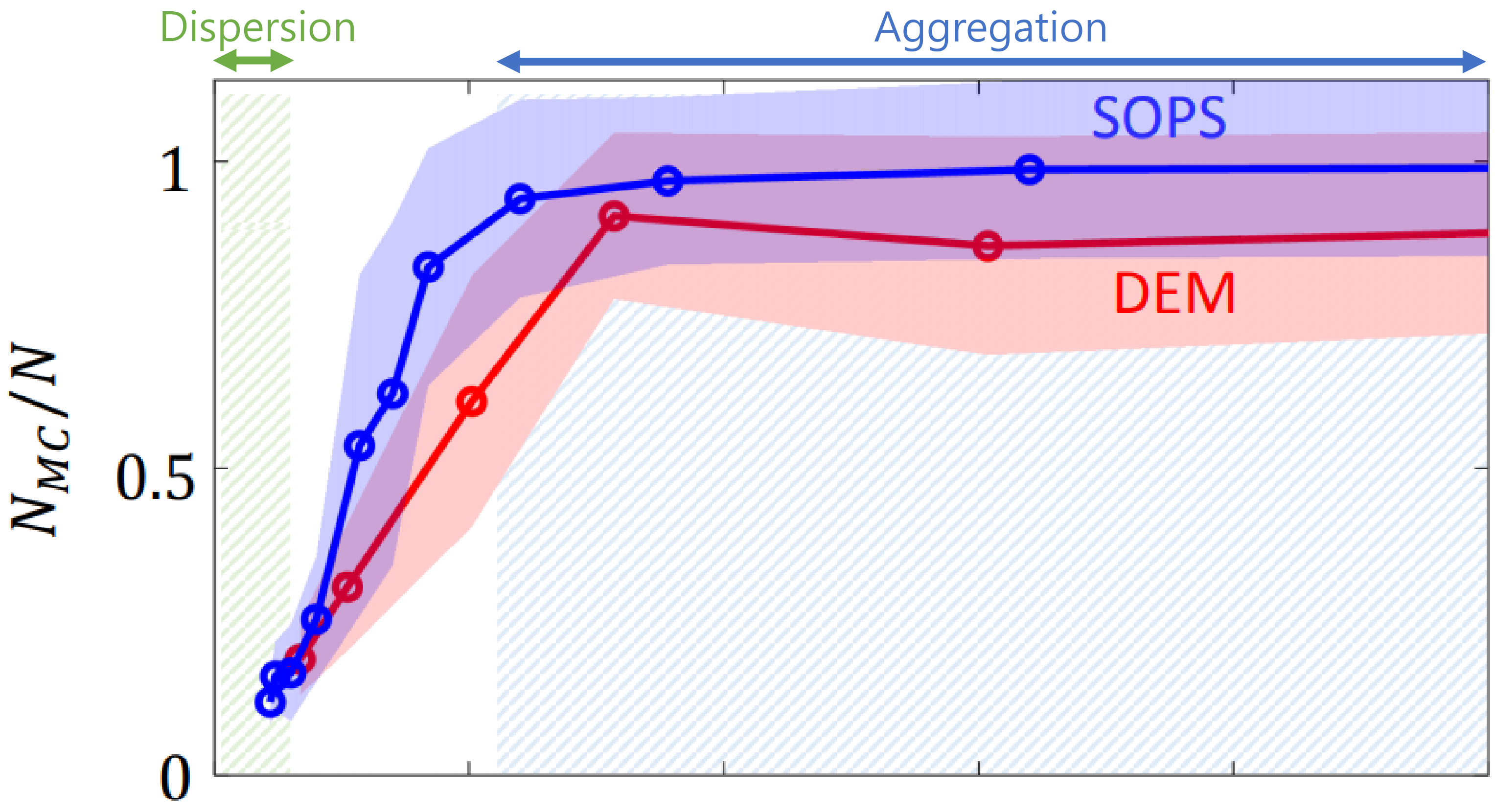
BOBbots: Aggregation and Dispersion

We map F_M to λ in experiments to obtain $\lambda_{eff} = \exp(\beta F_M)$, where β is inverse temperature, yielding agreement between the theoretical predictions and the empirical data.



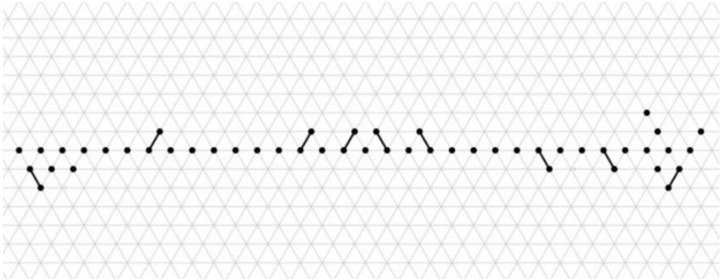
[5] Li, Dutta, Cannon, **Daymude**, Avinery, Aydin, Richa, Goldman, Randall. “Programming Active Cohesive Granular Matter with Mechanically Induced Phase Changes.” **Science Advances**, 2021.

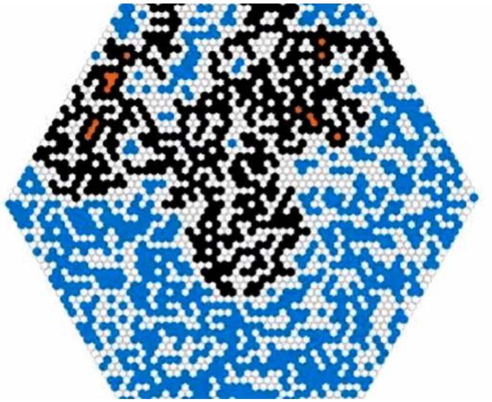
Summary: Aggregation for Analog Robots

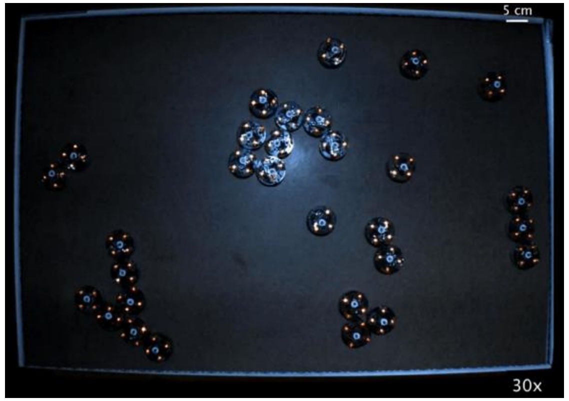
How can digital algorithms for collective behavior be translated to simple, analog systems?

Connect biased random decisions to the physics of local interactions.

Markov chain $\mathcal{M}_C$ for Compression → Markov chain $\mathcal{M}_A$ for Aggregation → Aggregation in BOBbots



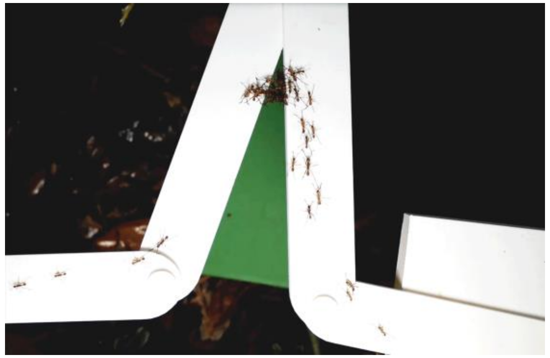




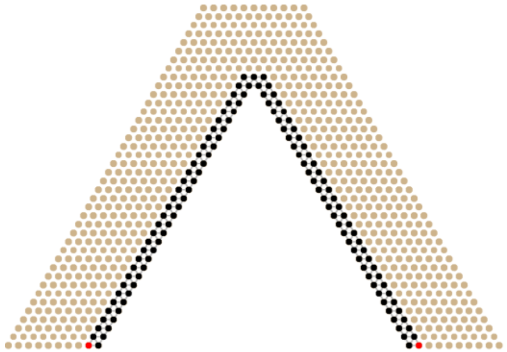
Bridging Algorithmic Theory to Physical Mechanics

Design distributed algorithms that <u>leverage</u> equilibrium statistical physics to quantitatively capture and predict the nonequilibrium dynamics of living and analog systems.

<u>Shortcut Bridging</u>



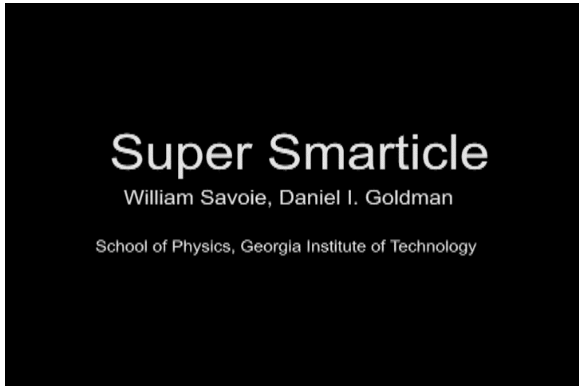
Eciton army ants
RLPKCG 2015

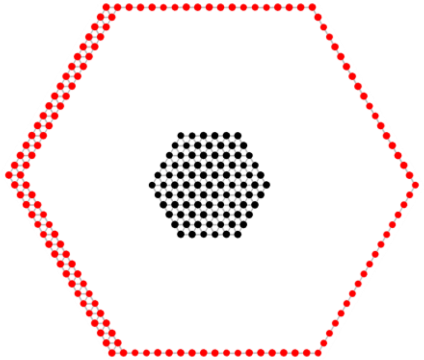


Distributed, Stochastic Algorithm

<u>Directed Locomotion</u>

"Supersmarticle" Collectives





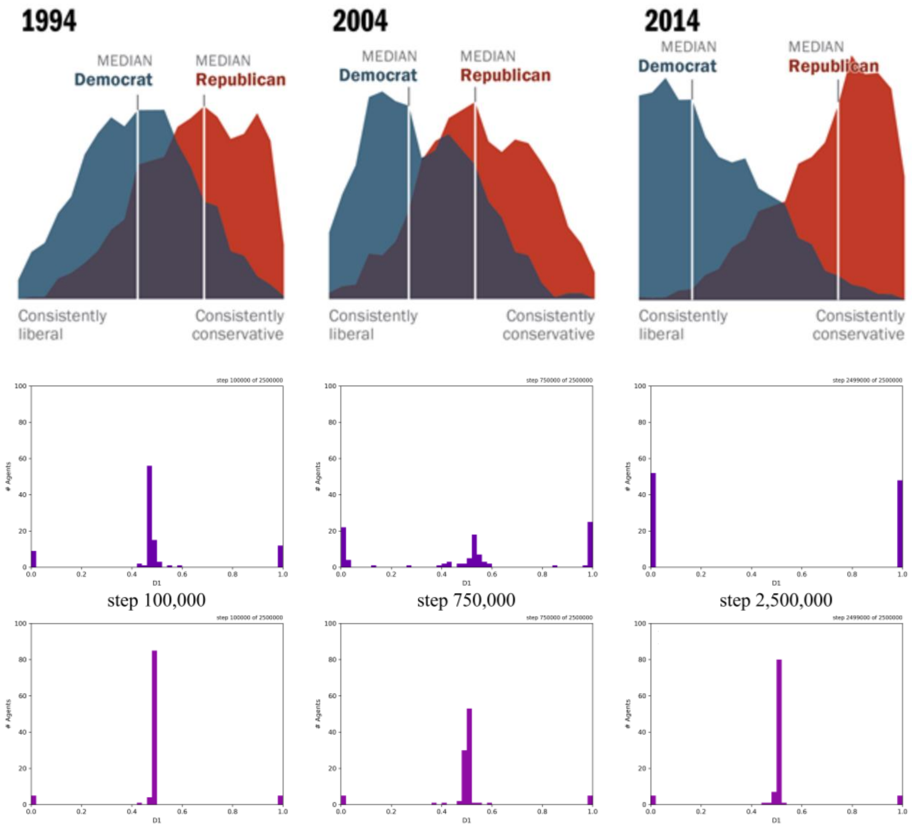
[7] Andrés Arroyo, Cannon, **Daymude**, Randall, Richa. "A Stochastic Approach to Shortcut Bridging in Programmable Matter." **Natural Computing**, 2018.
[8] Savoie, Cannon, **Daymude**, Warkentin, Li, Richa, Randall, Goldman. "Phototactic Supersmarticles." **Artificial Life and Robotics**, 2018.

Characterizing Biological and Social Complex Systems

Use formal distributed modeling to characterize observed emergent phenomena in complex systems as a function of the local interactions that produce them.

My recent work has modeled and analyzed the dynamics of **political polarization**.

[Pew Research Center]

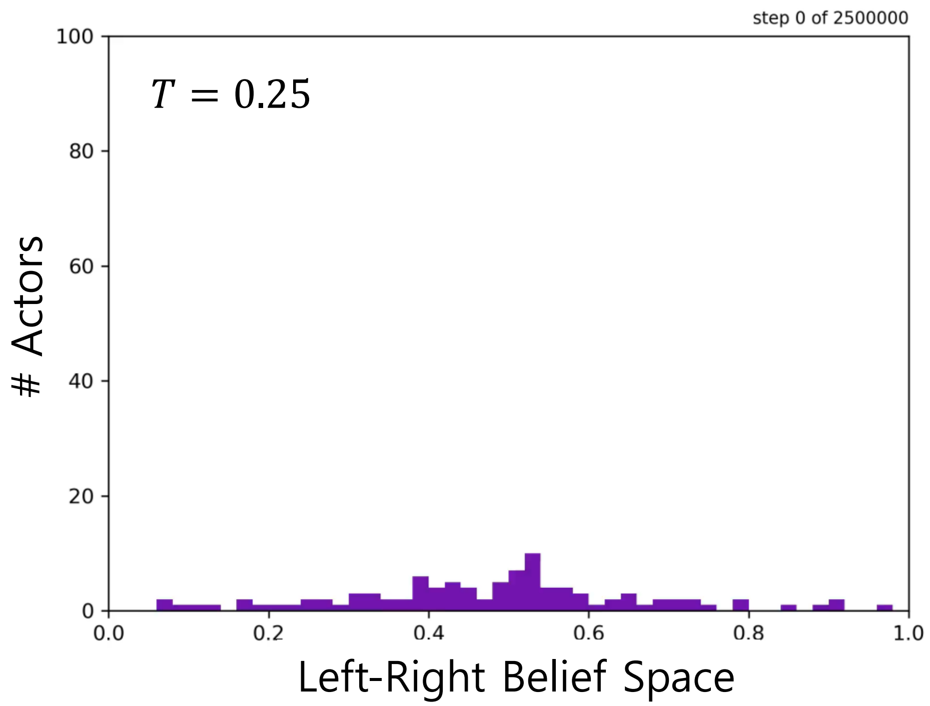


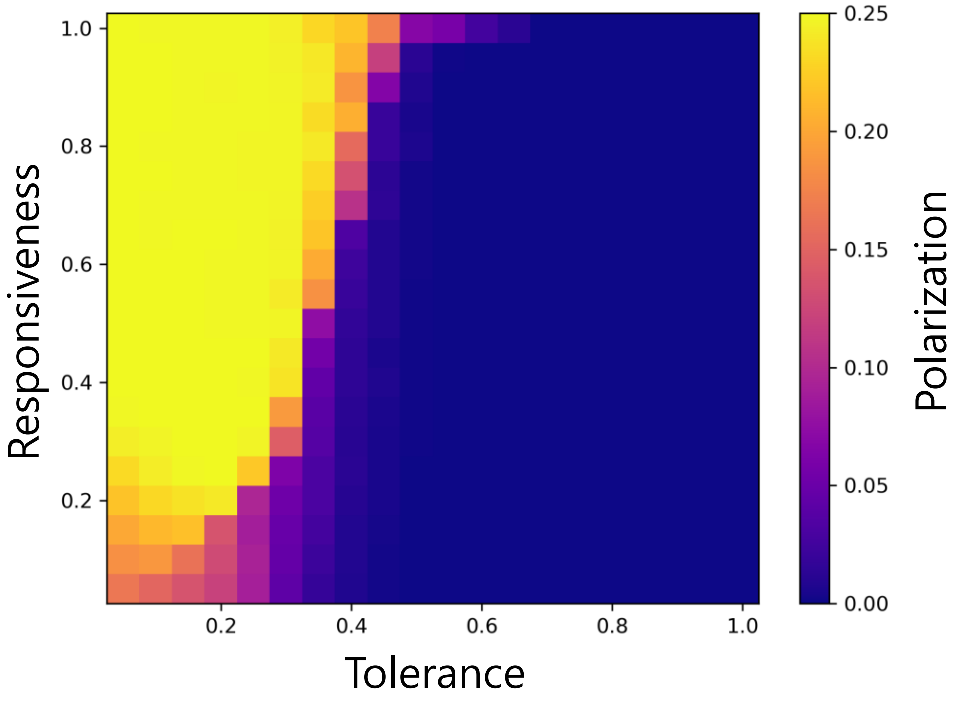
[9] Axelrod, **Daymude**, Forrest. "Preventing Extreme Polarization of Political Attitudes." To appear in **PNAS**, 2021.

Characterizing Biological and Social Complex Systems

Our **Attraction-Repulsion Model (ARM)** has two simple rules:

1. Actors tend to be exposed to views similar to their own (exposure w.p. $(1/2)^{d/E}$).
2. Interaction between similar actors (within tolerance T) reduces their ideological difference (by a fraction R), while interaction between dissimilar actors increases their difference.



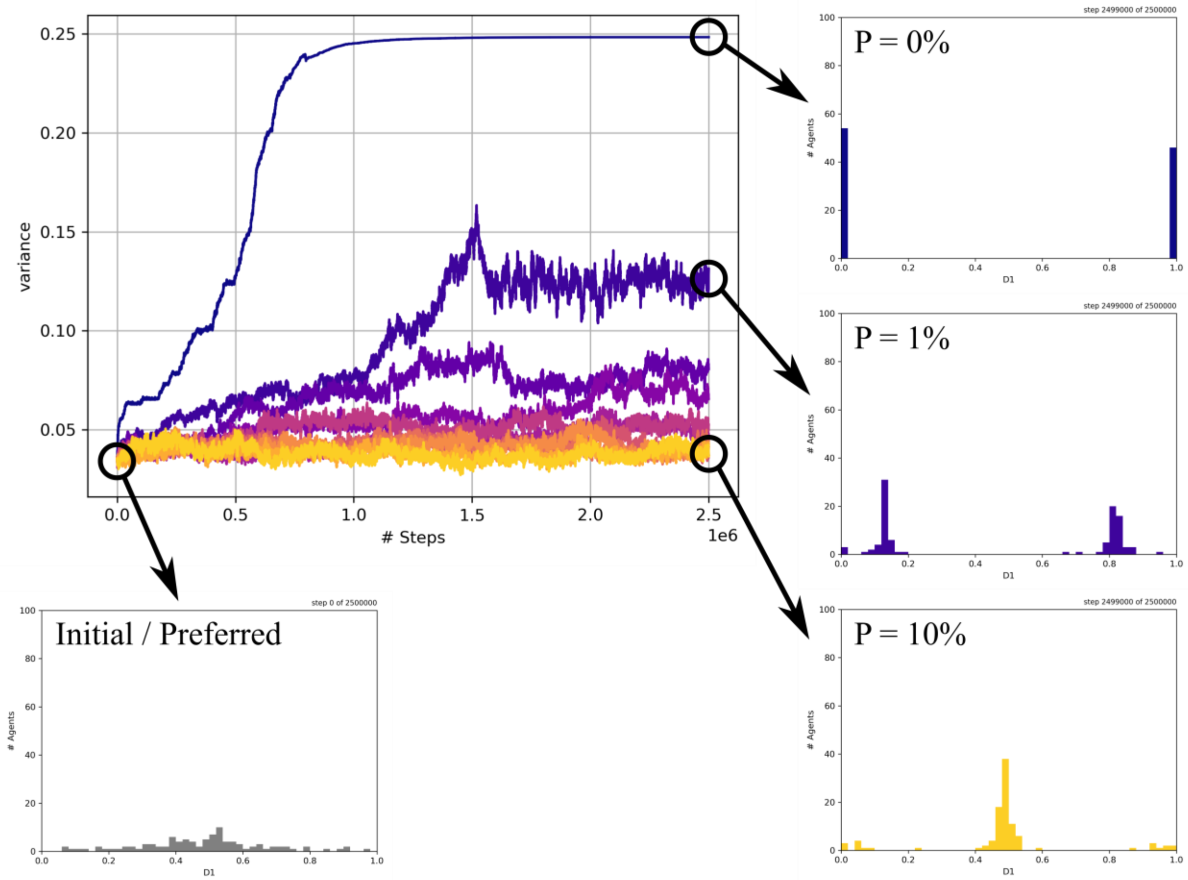


[9] Axelrod, **Daymude**, Forrest. "Preventing Extreme Polarization of Political Attitudes." To appear in **PNAS**, 2021.

Characterizing Biological and Social Complex Systems

The ARM suggests that extreme polarization can be mitigated with:

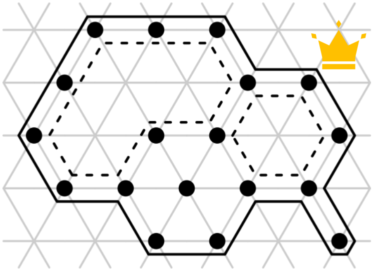
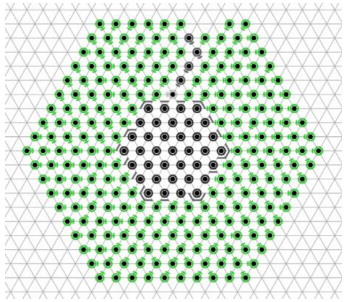
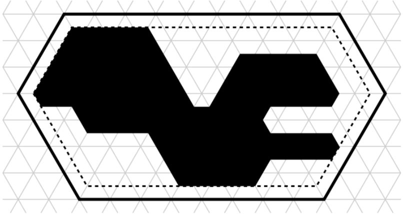
- High tolerance T (leads to consensus).
- Low exposure E (encourages insulated communities).
- Any self-interest for a moderate position (leads to bimodal or centrist distributions).
- Very early or very strong external shock (leads to consensus).



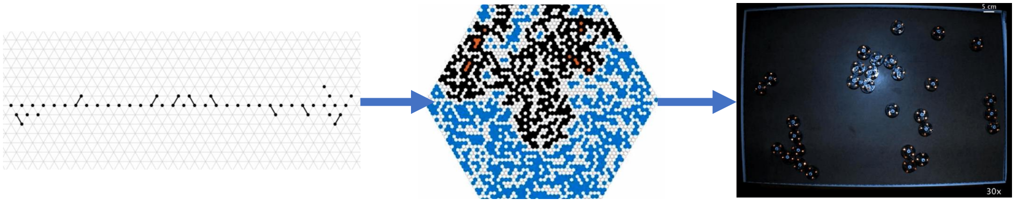
[9] Axelrod, **Daymude**, Forrest. "Preventing Extreme Polarization of Political Attitudes." To appear in **PNAS**, 2021.

Conclusion

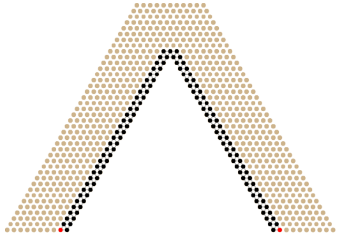
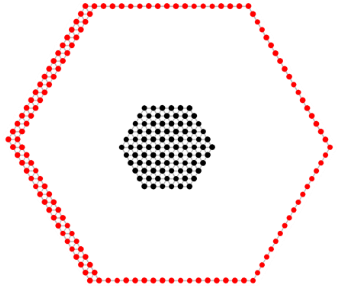
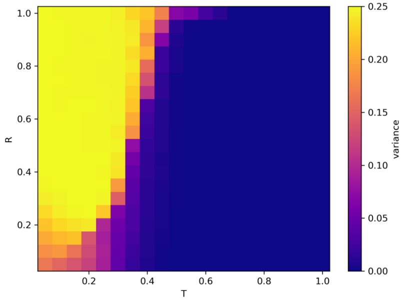
1. Constant-size memory, local communication, and local movements suffice to program many desired behaviors in digital collectives.

2. By connecting biased random decisions to the physics of local interactions, we can translate digital algorithms driving emergent collective behavior for simple analog robots.



3. This same framework can suggest local rules and decisions that capture observed collective behavior in biological and social complex systems.

Thank you!

jdaymude.github.io

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